

2005 AP extended response

calc calc

- #2 a) *Note* - I was initially confused as I interpreted this to mean what the net change was which would be

$$\int_0^7 f(t) dt - \int_0^7 g(t) dt$$

↑ ↑
enters exits

but realized the word used was 'entered' and focused solely on $f(t)$

∴ (therefore)

Answer (with calc)

$$\int_0^7 100t^2 \sin(\sqrt{t}) = 8263.8$$

*notice full sentence answer WITH units

8264 gallons enter in first 7 hours

b) *graphical analysis*

adjust windows appropriately

found the intersection of $f(t)$ and $y=250$
and $f(t)$ and $y=2000$

1st intersection is $x=1.617$ hours

prior ... or 0 hours to 1.617 hours is decreasing because the water exits faster than it enters.

(cont'd)

2nd intersection is: $x = 5.076$

The section of graph from hour 3 to 5.076 shows the incoming water is less than the outgoing.

So decreasing from 0 to 1.617 and 3 to 5.076

HA!
realized they gave us points
for AFTER
I did the work

c) Greatest will either be at $t=0$ or $t=3$ or $t=7$

(only $t=3$ is where graph goes from increasing to decreasing)

<u>time</u>	<u>amount</u>
$t=0$	5000
$t=3$	$5000 + \int_0^3 100t^2 \sinh(\sqrt{t}) dt - \int_0^3 200 dt$ 5126.591
$t=7$	$5126.591 + \int_3^7 100t^2 \sinh(\sqrt{t}) dt - \int_3^7 200 dt$ 4513.807

The water amount is greatest at time of 3 hours. There is 5126.591 gallons at this time.

Calc. OK! no part d ... not learning inverse functions until chp 5.

③

a) $h(1) = f(g(1)) - 6 \Rightarrow 1 - 6 \Rightarrow -5$

$h(3) = f(g(3)) - 6 \Rightarrow -1 - 6 \Rightarrow -7$

since graph is differentiable for all real numbers and since

$-7 < -5 < 1$

this guarantees there is an

$3 < c < 1$

attempted
rechecked
answer
key
explicitly
stated
Intermediate
Value
Theorem.

so awesome
if you
remember it!

b) Average rate of change from
1 to 3 is $\frac{h(3) - h(1)}{3 - 1}$

$$\frac{-7 - (-5)}{3 - 1} = \frac{-2}{2} = -1$$

A theorem states if the function is differentiable at all x 's and the average rate of change for an interval is -1 , guarantees there will be a point whose instantaneous rate of change will be -1 too.

would have
if saved time
then I remember
Theorem name.

can't d

$$c) \quad w(x) = \int_1^{g(x)} f(t) dt$$

$$\frac{d}{dx} w(x) = \frac{d}{dx} \int_1^{g(x)} f(t) dt$$

$$w'(x) = f(g(x)) \cdot g'(x)$$

$$w'(3) = f(\underbrace{g(3)}_4) \cdot \underbrace{g'(3)}_{-1}$$
$$= f(4) \cdot (-1)$$
$$= 2 \cdot (-1)$$

$$\boxed{w'(3) = -2}$$

5) a) radius = 30 ft at $t=5$ (r!t)
rate of change of radius at $t=5$ is 2.0

$$\text{so } r = 30 + 2(x-5)$$

$$r = 30 + 2(5.4-5)$$

$$r = 30 + 2(0.4)$$

$$r = 30 + 0.8$$

$$r = 30.8$$

30.8 feet is estimate... this is overestimation since graph is concave down... the rate of change will actually be decreasing as time advances.

$$b) \frac{d}{dt} [V] = \frac{d}{dt} \left[\frac{4}{3} \pi r^3 \right]$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\text{at } t=5$$

$$r=30$$

$$\frac{dr}{dt} = 2.0$$

$$\frac{dV}{dt} = 4\pi (30)^2 (2) \quad \frac{\text{ft}^3}{\text{min}}$$

★ Since non-calc... I could stop here...
this would be full credit as it is equivalent to the fully computed approx
7700 ft^3/min

c) right method

ht = width

$$4(2) + 2(3) + 1.2(2) + 0.6(4) + 0.5(1)$$

$$8 + 6 + 2.4 + 2.4 + 0.5$$

$$14 + 4.8 + 0.5$$

19.3 feet

this is the increase in the radius of the balloon in the 12 seconds (approximate anyway)

19.3 feet

d. This is an underestimate, as right hand methods will underestimate decreasing graphs.

