2008 AP Calculus AB Multiple Choice Exam

Section 1 No Calculator Active

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Section 2 Calculator Active

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1.
$$\lim_{x \to \infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$$
 is
(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

$$2. \int \frac{1}{x^2} dx =$$

(A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$

3. If
$$f(x) = (x-1)(x^2+2)^3$$
, then $f'(x) =$
(A) $6x(x^2+2)^2$
(B) $6x(x-1)(x^2+2)^2$
(C) $(x^2+2)^2(x^2+3x-1)$
(D) $(x^2+2)^2(7x^2-6x+2)$
(E) $-3(x-1)(x^2+2)^2$

4.
$$\int (\sin(2x) + \cos(2x)) dx =$$

(A) $\frac{1}{2} \cos(2x) + \frac{1}{2} \sin(2x) + C$
(B) $-\frac{1}{2} \cos(2x) + \frac{1}{2} \sin(2x) + C$
(C) $2 \cos(2x) + 2 \sin(2x) + C$
(D) $2 \cos(2x) - 2 \sin(2x) + C$
(E) $-2 \cos(2x) + 2 \sin(2x) + C$

5.
$$\lim_{x \to 0} \frac{5x^4 + 8x^2}{3x^4 - 16x^2}$$
 is
(A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{5}{3} + 1$ (E) nonexistent

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2\\ 1 & \text{if } x = 2 \end{cases}$$

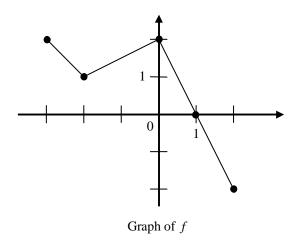
6. Let f be the function defined above. Which of the following statements about f are true?

- I. *f* has a limit at x = 2.
- II. f is continuous at x = 2.
- III. f is differentiable at x = 2.
- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) I, II, and III

7. A particle moves along the *x*-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \ge 0$. If the particle is at position x = 2 at time t = 0, what is the position of the particle at t = 1?

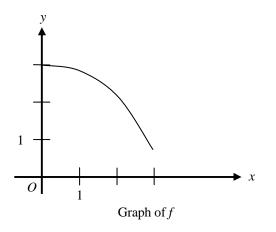
(A) 4 (B) 6 (C) 9 (D) 11 (E) 12

8. If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$ (A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$



9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t) dt$, which of the following values is greatest?

(A) g(-3) (B) g(-2) (C) g(0) (D) g(1) (E) g(2)



10. The graph of function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

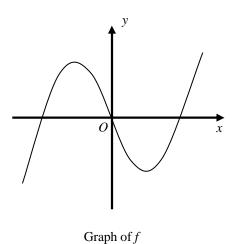
(A) $\int_{1}^{3} f(x) dx$

(B) Left Riemann sum approximation of $\int_{1}^{3} f(x) dx$ with 4 subintervals of equal length

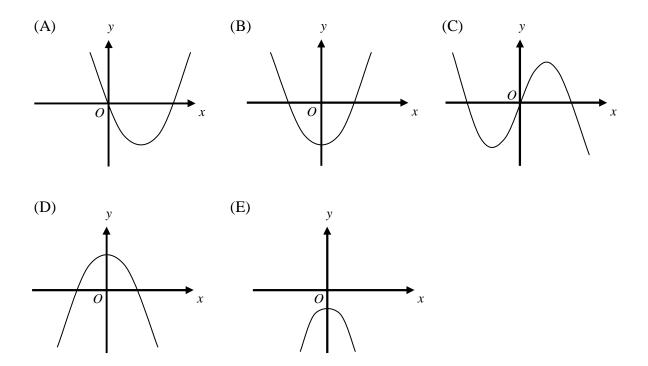
(C) Right Riemann sum approximation of $\int_{1}^{3} f(x) dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_{1}^{3} f(x) dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_{1}^{3} f(x) dx$ with 4 subintervals of equal length



11. The graph of a function f is shown above. Which of the following could be the graph of f, the derivative of f?



12. If
$$f(x) = e^{(2/x)}$$
, then $f'(x) =$
(A) $2e^{(2/x)} \ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

13. If
$$f(x) = x^2 + 2x$$
, then $\frac{d}{dx} (f(\ln x)) =$
(A) $\frac{2\ln x + 2}{x}$ (B) $2x\ln x + 2$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$ (E) $\frac{2x+2}{x}$

AP Calculus 2008 Multiple Choice

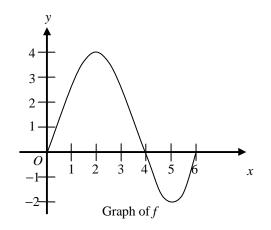
x	0	1	2	3
f''(x)	5	0	—7	4

- 14. The polynomial function f has selected values of its second derivative f " given in the table above. Which of the following statements must be true?
 - (A) f is increasing on the interval (0, 2).
 - (B) f is decreasing on the interval (0, 2).
 - (C) f has a local maximum at x = 1.
 - (D) The graph of f has a point of inflection at x = 1.
 - (E) The graph of f changes concavity in the interval (0, 2).

15.
$$\int \frac{x}{x^2 - 4} dx =$$
(A)
$$\frac{-1}{4(x^2 - 4)^2} + C$$
(B)
$$\frac{1}{2(x^2 - 4)} + C$$
(C)
$$\frac{1}{2} \ln |x^2 - 4| + C$$
(D)
$$2 \ln |x^2 - 4| + C$$
(E)
$$\frac{1}{2} \arctan \left(\frac{x}{2} \right) + C$$

1

16. If
$$\sin(xy) = x$$
, then $\frac{dy}{dx} =$
(A) $\frac{1}{\cos(xy)}$
(B) $\frac{1}{x\cos(xy)}$
(C) $\frac{1-\cos(xy)}{\cos(xy)}$
(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$
(E) $\frac{y(1-\cos(xy))}{x}$



17. The graph of the function f shown above has horizontal tangents at x = 2 and x = 5. Let g be the function defined by $g(x) = \int_0^x f(t) dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

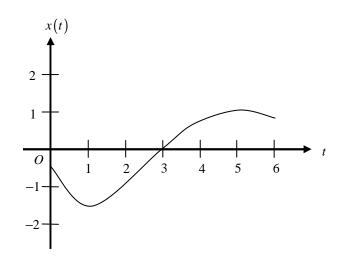
18. In the *xy*-plane, the line x + y = k, where *k* is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of *k*?

$$(A) -3 (B) -2 (C) -1 (D) 0 (E) 1$$

19. What are all horizontal asymptotes of the graph of $y = \frac{5+2^x}{1-2^x}$ in the *xy*-plane?

- (A) y = -1 only
- (B) y = 0 only
- (C) y = 5 only
- (D) y = -1 and y = 0
- (E) y = -1 and y = 5

- 20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?
 - (A) 0 only
 - (B) 3 only
 - (C) 0 and 6 only
 - (D) 3 and 6 only
 - (E) 0, 3, and 6



- 21. A particle moves along a straight line. The graph of the particle's position x(t) at time t is shown above for 0 < t < 6. The graph has horizontal tangents at t = 1 and t = 5 and a point of inflection at t = 2. For what values of t is the velocity of the particle increasing?
 - (A) 0 < t < 2
 - (B) 1 < t < 5
 - (C) 2 < t < 6
 - (D) 3 < t < 5 only
 - (E) 1 < t < 2 and 5 < t < 6

AP Calculus 2008 Multiple Choice

22. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

(A)
$$\frac{dp}{dt} = kp$$

(B) $\frac{dp}{dt} = kp(N-p)$
(C) $\frac{dp}{dt} = kp(p-N)$
(D) $\frac{dp}{dt} = kt(N-t)$
(E) $\frac{dp}{dt} = kt(t-N)$

- 23. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition y(3) = -2?
 - (A) $y = 2e^{-9+x^3/3}$ (B) $y = -2e^{-9+x^3/3}$ (C) $y = \sqrt{\frac{2x^3}{3}}$ (D) $y = \sqrt{\frac{2x^3}{3} - 14}$ (E) $y = -\sqrt{\frac{2x^3}{3} - 14}$

24. The function f is twice differentiable with f(2)=1, f'(2)=4, and f''(2)=3. What is the value of the approximation of f(1.9) using the line tangent to the graph of f at x=2?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

$$f(x) = \begin{cases} cx+d & \text{for } x \le 2\\ x^2 - cx & \text{for } x > 2 \end{cases}$$

- 25. Let f be the function defined above, where c and d are constants. If f is differentiable at x = 2, what is the value of c + d?
 - (A) -4 (B) -2 (C) 0 (D) 2 (E) 4

26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which

 $x = \frac{1}{4}?$

(A) 2 (B)
$$\frac{1}{2}$$
 (C) 0 (D) $-\frac{1}{2}$ (E) -2

27. Shown above is a slope field for which of the following differential equations?

(A)
$$\frac{dy}{dx} = xy$$

(B) $\frac{dy}{dx} = xy - y$
(C) $\frac{dy}{dx} = xy + y$
(D) $\frac{dy}{dx} = xy + x$
(E) $\frac{dy}{dx} = (x+1)^3$

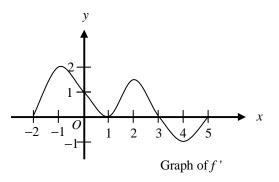
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28. Let f be a differentiable function such that f(3) = 15, f(6) = 3, f'(3) = -8, and f'(6) = -2. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of g'(3)?

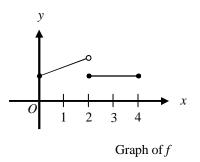
(A)
$$-\frac{1}{2}$$

(B) $-\frac{1}{8}$
(C) $\frac{1}{6}$
(D) $\frac{1}{3}$

(E) The value of g'(3) cannot be determined from the information given.



- 76. The graph of f', the derivative f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?
 - (A) [-2, 1] only
 - (B) [-2, 3]
 - (C) [3, 5] only
 - (D) [0, 1.5] and [3, 5]
 - (E) [-2, -1], [1, 2], and [4, 5]



- 77. The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?
 - I. $\lim_{x \to 2^{-}} f(x)$ exists. II. $\lim_{x \to 2^{+}} f(x)$ exists. III. $\lim_{x \to 2^{+}} f(x)$ exists. (A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

- 78. The first derivative of the function f is defined by $f'(x) = \sin(x^3 x)$ for $0 \le x \le 2$. On what interval(s) is f increasing?
 - (A) $1 \le x \le 1.445$
 - (B) $1 \le x \le 1.691$
 - (C) $1.445 \le x \le 1.875$
 - (D) $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$
 - (E) $0 \le x \le 1$ and $1.691 \le x \le 2$

79. If
$$\int_{-5}^{2} f(x) dx = -17$$
 and $\int_{5}^{2} f(x) dx = -4$, what is the value of $\int_{-5}^{5} f(x) dx$?
(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

80. The derivative of the function f is given by $f'(x) = x^2 \cos(x^2)$. How many points of inflection does the graph of f have on the open interval (-2, 2)?

(A) One (B) Two (C) Three (D) Four (E) Five

81. If G(x) is an antiderivative for f(x) and G(2) = -7, then G(4) =

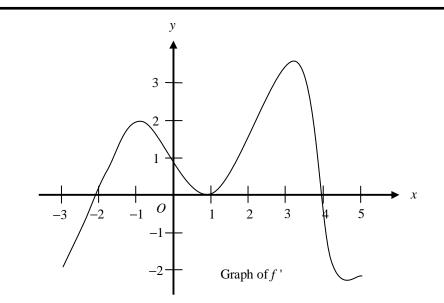
(A) f'(4)(B) -7 + f'(4)(C) $\int_{2}^{4} f(t) dt$ (D) $\int_{2}^{4} (-7 + f(t)) dt$ (E) $-7 + \int_{2}^{4} f(t) dt$

82. A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \ge 0$. What is the acceleration of the particle at time t = 3?

(A) -0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

83. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and y = x + 5?

- (A) 10.667
 (B) 11.833
 (C) 14.583
 (D) 21.333
- (E) 32



- 84. The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at x = -1, x = 1, and x = 3. At which of the following values of x does f have a relative maximum?
 - (A) -2 only
 - (B) 1 only
 - (C) 4 only
 - (D) -1 and 3 only
 - (E) -2, 1, and 4

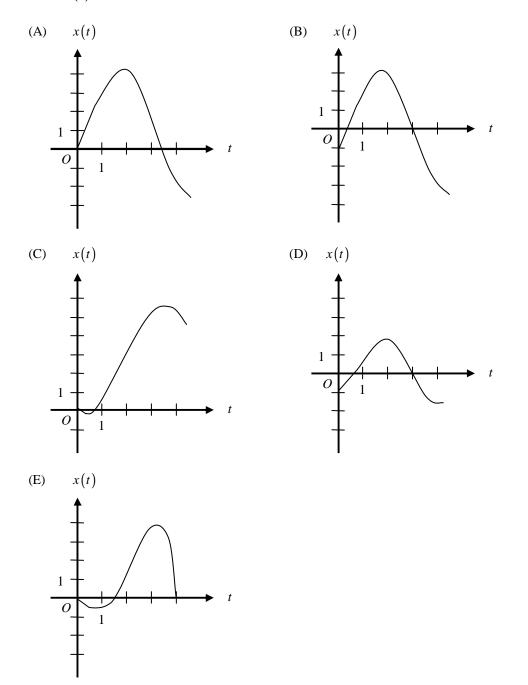
x	-4	-3	-2	-1
f(x)	0.75	-1.5	-2.25	-1.5
f'(x)	-3	-1.5	0	1.5

85. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval [-4, -1], what is the value of $\int_{-4}^{-1} f'(x) dx$?

(A) -4.5 (B) -2.25 (C) 0 (D) 2.25 (E) 4.5

t	0	1	2	3	4
v(t)	-1	2	3	0	-4

86. The table gives selected values of the velocity, v(t), of a particle moving along the *x*-axis. At time t = 0, the particle is at the origin. Which of the following could be the graph of the position, x(t), of the particle for $0 \le t \le 4$?



87. An object traveling in a straight line has position x(t) at time *t*. If the initial position is x(0) = 2 and the velocity of the object is $v(t) = \sqrt[3]{1+t^2}$, what is the position of the object at time t = 3?

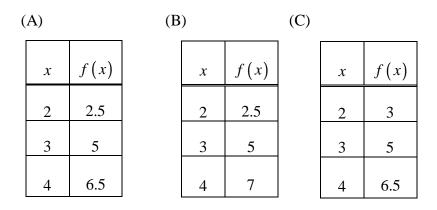
(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

88. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area *S* of a sphere with radius *r* is $S = 4\pi r^2$)

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

89. The function f is continuous for $-2 \le x \le 2$ and f(-2) = f(2) = 0. If there is no c, where -2 < c < 2, for which f'(c) = 0, which of the following statements must be true?

- (A) For -2 < k < 2, f'(k) > 0.
- (B) For -2 < k < 2, f'(k) < 0.
- (C) For -2 < k < 2, f'(k) exists.
- (D) For -2 < k < 2, f'(k) exists, but f' is not continuous.
- (E) For some k, where -2 < k < 2, f'(k) does not exist.
- 90. The function f is continuous on the closed interval [2, 4] and twice differentiable on the open interval (2, 4). If f'(3) = 2 and f''(x) < 0 on the open interval (2, 4), which of the following could be a table of values for f?



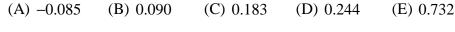
(D)

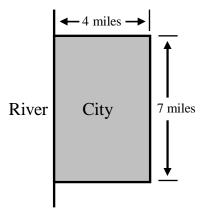
(E)

x	f(x)	x	f(x)
2	3	2	3.5
3	5	3	5
4	7	4	7.5

AP Calculus 2008 Multiple Choice

91. What is the average value of $y = \frac{\cos x}{x^2 + x + 2}$ on the closed interval [-1, 3]?





92. A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is f(x) persons per square mile. Which of the following expressions gives the population of the city?

(A) $\int_{0}^{4} f(x) dx$ (B) $7 \int_{0}^{4} f(x) dx$ (C) $28 \int_{0}^{4} f(x) dx$ (D) $\int_{0}^{7} f(x) dx$ (E) $4 \int_{0}^{7} f(x) dx$