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#  Calculus AB Multiple Choice Exam <br> <br> Section 1 <br> <br> Section 1 <br> No Calculator Active 

#  Calculus AB Multiple Choice Exam <br> Section 2 Calculator Active 

AP Calculus 2008 Multiple Choice

1. $\lim _{x \rightarrow \infty} \frac{(2 x-1)(3-x)}{(x-1)(x+3)}$ is
(A) -3
(B) -2
(C) 2
(D) 3
(E) nonexistent
2. $\int \frac{1}{x^{2}} d x=$
(A) $\ln x^{2}+C$
(B) $-\ln x^{2}+C$
(C) $x^{-1}+C$
(D) $-x^{-1}+C$
(E) $-2 x^{-3}+C$

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3. If $f(x)=(x-1)\left(x^{2}+2\right)^{3}$, then $f^{\prime}(x)=$
(A) $6 x\left(x^{2}+2\right)^{2}$
(B) $6 x(x-1)\left(x^{2}+2\right)^{2}$
(C) $\left(x^{2}+2\right)^{2}\left(x^{2}+3 x-1\right)$
(D) $\left(x^{2}+2\right)^{2}\left(7 x^{2}-6 x+2\right)$
(E) $-3(x-1)\left(x^{2}+2\right)^{2}$
4. $\int(\sin (2 x)+\cos (2 x)) d x=$
(A) $\frac{1}{2} \cos (2 x)+\frac{1}{2} \sin (2 x)+C$
(B) $-\frac{1}{2} \cos (2 x)+\frac{1}{2} \sin (2 x)+C$
(C) $2 \cos (2 x)+2 \sin (2 x)+C$
(D) $2 \cos (2 x)-2 \sin (2 x)+C$
(E) $-2 \cos (2 x)+2 \sin (2 x)+C$
5. $\lim _{x \rightarrow 0} \frac{5 x^{4}+8 x^{2}}{3 x^{4}-16 x^{2}}$ is
(A) $-\frac{1}{2}$
(B) 0
(C) 1
(D) $\frac{5}{3}+1$
(E) nonexistent

$$
f(x)= \begin{cases}\frac{x^{2}-4}{x-2} & \text { if } x \neq 2 \\ 1 & \text { if } x=2\end{cases}
$$

6. Let $f$ be the function defined above. Which of the following statements about $f$ are true?
I. $f$ has a limit at $x=2$.
II. $f$ is continuous at $x=2$.
III. $f$ is differentiable at $x=2$.
(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III
7. A particle moves along the $x$-axis with velocity given by $v(t)=3 t^{2}+6 t$ for time $t \geq 0$. If the particle is at position $x=2$ at time $t=0$, what is the position of the particle at $t=1$ ?
(A) 4
(B) 6
(C) 9
(D) 11
(E) 12
8. If $f(x)=\cos (3 x)$, then $f^{\prime}\left(\frac{\pi}{9}\right)=$
(A) $\frac{3 \sqrt{3}}{2}$
(B) $\frac{\sqrt{3}}{2}$
(C) $-\frac{\sqrt{3}}{2}$
(D) $-\frac{3}{2}$
(E) $-\frac{3 \sqrt{3}}{2}$


Graph of $f$
9. The graph of the piecewise linear function $f$ is shown in the figure above. If $g(x)=\int_{-2}^{x} f(t) d t$, which of the following values is greatest?
(A) $g(-3)$
(B) $g(-2)$
(C) $g(0)$
(D) $g(1)$
(E) $g(2)$

10. The graph of function $f$ is shown above for $0 \leq x \leq 3$. Of the following, which has the least value?
(A) $\int_{1}^{3} f(x) d x$
(B) Left Riemann sum approximation of $\int_{1}^{3} f(x) d x$ with 4 subintervals of equal length
(C) Right Riemann sum approximation of $\int_{1}^{3} f(x) d x$ with 4 subintervals of equal length
(D) Midpoint Riemann sum approximation of $\int_{1}^{3} f(x) d x$ with 4 subintervals of equal length
(E) Trapezoidal sum approximation of $\int_{1}^{3} f(x) d x$ with 4 subintervals of equal length


Graph of $f$
11. The graph of a function $f$ is shown above. Which of the following could be the graph of $f^{\prime}$, the derivative of $f$ ?
(A)

(B)



(E)


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12. If $f(x)=e^{(2 / x)}$, then $f^{\prime}(x)=$
(A) $2 e^{(2 / x)} \ln x$
(B) $e^{(2 / x)}$
(C) $e^{\left(-2 / x^{2}\right)}$
(D) $-\frac{2}{x^{2}} e^{(2 / x)}$
(E) $-2 x^{2} e^{(2 / x)}$
13. If $f(x)=x^{2}+2 x$, then $\frac{d}{d x}(f(\ln x))=$
(A) $\frac{2 \ln x+2}{x}$
(B) $2 x \ln x+2$
(C) $2 \ln x+2$
(D) $2 \ln x+\frac{2}{x}$
(E) $\frac{2 x+2}{x}$

| $x$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $f^{\prime \prime}(x)$ | 5 | 0 | -7 | 4 |

14. The polynomial function $f$ has selected values of its second derivative $f$ " given in the table above. Which of the following statements must be true?
(A) $f$ is increasing on the interval $(0,2)$.
(B) $f$ is decreasing on the interval $(0,2)$.
(C) $f$ has a local maximum at $x=1$.
(D) The graph of $f$ has a point of inflection at $x=1$.
(E) The graph of $f$ changes concavity in the interval $(0,2)$.
15. $\int \frac{x}{x^{2}-4} d x=$
(A) $\frac{-1}{4\left(x^{2}-4\right)^{2}}+C$
(B) $\frac{1}{2\left(x^{2}-4\right)}+C$
(C) $\frac{1}{2} \ln \left|x^{2}-4\right|+C$
(D) $2 \ln \left|x^{2}-4\right|+C$
(E) $\frac{1}{2} \arctan \left(\frac{x}{2}\right)+C$
16. If $\sin (x y)=x$, then $\frac{d y}{d x}=$
(A) $\frac{1}{\cos (x y)}$
(B) $\frac{1}{x \cos (x y)}$
(C) $\frac{1-\cos (x y)}{\cos (x y)}$
(D) $\frac{1-y \cos (x y)}{x \cos (x y)}$
(E) $\frac{y(1-\cos (x y))}{x}$

17. The graph of the function $f$ shown above has horizontal tangents at $x=2$ and $x=5$. Let $g$ be the function defined by $g(x)=\int_{0}^{x} f(t) d t$. For what values of $x$ does the graph of $g$ have a point of inflection?
(A) 2 only
(B) 4 only
(C) 2 and 5 only
(D) 2, 4, and 5
(E) 0,4 , and 6
18. In the $x y$-plane, the line $x+y=k$, where $k$ is a constant, is tangent to the graph of $y=x^{2}+3 x+1$. What is the value of $k$ ?
(A) -3
(B) -2
(C) -1
(D) 0
(E) 1
19. What are all horizontal asymptotes of the graph of $y=\frac{5+2^{x}}{1-2^{x}}$ in the $x y$-plane?
(A) $y=-1$ only
(B) $y=0$ only
(C) $y=5$ only
(D) $y=-1$ and $y=0$
(E) $y=-1$ and $y=5$
20. Let $f$ be a function with a second derivative given by $f^{\prime \prime}(x)=x^{2}(x-3)(x-6)$. What are the $x$-coordinates of the points of inflection of the graph of $f$ ?
(A) 0 only
(B) 3 only
(C) 0 and 6 only
(D) 3 and 6 only
(E) 0, 3, and 6

21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time $t$ is shown above for $0<t<6$. The graph has horizontal tangents at $t=1$ and $t=5$ and a point of inflection at $t=2$. For what values of $t$ is the velocity of the particle increasing?
(A) $0<t<2$
(B) $1<t<5$
(C) $2<t<6$
(D) $3<t<5$ only
(E) $1<t<2$ and $5<t<6$
22. A rumor spreads among a population of $N$ people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If $p$ denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time $t$, where $k$ is a positive constant?
(A) $\frac{d p}{d t}=k p$
(B) $\frac{d p}{d t}=k p(N-p)$
(C) $\frac{d p}{d t}=k p(p-N)$
(D) $\frac{d p}{d t}=k t(N-t)$
(E) $\frac{d p}{d t}=k t(t-N)$
23. Which of the following is the solution to the differential equation $\frac{d y}{d x}=\frac{x^{2}}{y}$ with the initial condition $y(3)=-2$ ?
(A) $y=2 e^{-9+x^{3} / 3}$
(B) $y=-2 e^{-9+x^{3} / 3}$
(C) $y=\sqrt{\frac{2 x^{3}}{3}}$
(D) $y=\sqrt{\frac{2 x^{3}}{3}-14}$
(E) $y=-\sqrt{\frac{2 x^{3}}{3}-14}$
24. The function $f$ is twice differentiable with $f(2)=1, f^{\prime}(2)=4$, and $f^{\prime \prime}(2)=3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of $f$ at $x=2$ ?
(A) 0.4
(B) 0.6
(C) 0.7
(D) 1.3
(E) 1.4

$$
f(x)=\left\{\begin{array}{lll}
c x+d & \text { for } & x \leq 2 \\
x^{2}-c x & \text { for } & x>2
\end{array}\right.
$$

25. Let $f$ be the function defined above, where $c$ and $d$ are constants. If $f$ is differentiable at $x=2$, what is the value of $c+d$ ?
(A) -4
(B) -2
(C) 0
(D) 2
(E) 4
26. What is the slope of the line tangent to the curve $y=\arctan (4 x)$ at the point at which $x=\frac{1}{4}$ ?
(A) 2
(B) $\frac{1}{2}$
(C) 0
(D) $-\frac{1}{2}$
(E) -2

27. Shown above is a slope field for which of the following differential equations?
(A) $\frac{d y}{d x}=x y$
(B) $\frac{d y}{d x}=x y-y$
(C) $\frac{d y}{d x}=x y+y$
(D) $\frac{d y}{d x}=x y+x$
(E) $\frac{d y}{d x}=(x+1)^{3}$
28. Let $f$ be a differentiable function such that $f(3)=15, f(6)=3, f^{\prime}(3)=-8$, and $f^{\prime}(6)=-2$. The function $g$ is differentiable and $g(x)=f^{-1}(x)$ for all $x$. What is the value of $g^{\prime}(3)$ ?
(A) $-\frac{1}{2}$
(B) $-\frac{1}{8}$
(C) $\frac{1}{6}$
(D) $\frac{1}{3}$
(E) The value of $g^{\prime}(3)$ cannot be determined from the information given.

29. The graph of $f^{\prime}$, the derivative $f$, is shown above for $-2 \leq x \leq 5$. On what intervals is $f$ increasing?
(A) $[-2,1]$ only
(B) $[-2,3]$
(C) $[3,5]$ only
(D) $[0,1.5]$ and $[3,5]$
(E) $[-2,-1],[1,2]$, and $[4,5]$


Graph of $f$
77. The figure above shows the graph of a function $f$ with domain $0 \leq x \leq 4$. Which of the following statements are true?
I. $\lim _{x \rightarrow 2^{-}} f(x)$ exists.
II. $\lim _{x \rightarrow 2^{+}} f(x)$ exists.
III. $\lim _{x \rightarrow 2} f(x)$ exists.
(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III
78. The first derivative of the function $f$ is defined by $f^{\prime}(x)=\sin \left(x^{3}-x\right)$ for $0 \leq x \leq 2$. On what interval(s) is $f$ increasing?
(A) $1 \leq x \leq 1.445$
(B) $1 \leq x \leq 1.691$
(C) $1.445 \leq x \leq 1.875$
(D) $0.577 \leq x \leq 1.445$ and $1.875 \leq x \leq 2$
(E) $0 \leq x \leq 1$ and $1.691 \leq x \leq 2$
79. If $\int_{-5}^{2} f(x) d x=-17$ and $\int_{5}^{2} f(x) d x=-4$, what is the value of $\int_{-5}^{5} f(x) d x$ ?
(A) -21
(B) -13
(C) 0
(D) 13
(E) 21
80. The derivative of the function $f$ is given by $f^{\prime}(x)=x^{2} \cos \left(x^{2}\right)$. How many points of inflection does the graph of $f$ have on the open interval $(-2,2)$ ?
(A) One
(B) Two
(C) Three
(D) Four
(E) Five
81. If $G(x)$ is an antiderivative for $f(x)$ and $G(2)=-7$, then $G(4)=$
(A) $f^{\prime}(4)$
(B) $-7+f^{\prime}(4)$
(C) $\int_{2}^{4} f(t) d t$
(D) $\int_{2}^{4}(-7+f(t)) d t$
(E) $-7+\int_{2}^{4} f(t) d t$
82. A particle moves along a straight line with velocity given by $v(t)=7-(1.01)^{-t^{2}}$ at time $t \geq 0$. What is the acceleration of the particle at time $t=3$ ?
(A) -0.914
(B) 0.055
(C) 5.486
(D) 6.086
(E) 18.087
83. What is the area enclosed by the curves $y=x^{3}-8 x^{2}+18 x-5$ and $y=x+5$ ?
(A) 10.667
(B) 11.833
(C) 14.583
(D) 21.333
(E) 32

84. The graph of the derivative of a function $f$ is shown in the figure above. The graph has horizontal tangent lines at $x=-1, x=1$, and $x=3$. At which of the following values of $x$ does $f$ have a relative maximum?
(A) -2 only
(B) 1 only
(C) 4 only
(D) -1 and 3 only
(E) $-2,1$, and 4

| $x$ | -4 | -3 | -2 | -1 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 0.75 | -1.5 | -2.25 | -1.5 |
| $f^{\prime}(x)$ | -3 | -1.5 | 0 | 1.5 |

85. The table above gives values of a function $f$ and its derivative at selected values of $x$. If $f^{\prime}$ is continuous on the interval $[-4,-1]$, what is the value of $\int_{-4}^{-1} f^{\prime}(x) d x$ ?
(A) -4.5
(B) -2.25
(C) 0
(D) 2.25
(E) 4.5

| $t$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $v(t)$ | -1 | 2 | 3 | 0 | -4 |

86. The table gives selected values of the velocity, $v(t)$, of a particle moving along the $x$-axis. At time $t=0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \leq t \leq 4$ ?
(A) $\quad x(t)$

(B) $\quad x(t)$

(C) $\quad x(t)$

(D) $x(t)$

(E) $\quad x(t)$

87. An object traveling in a straight line has position $x(t)$ at time $t$. If the initial position is $x(0)=2$ and the velocity of the object is $v(t)=\sqrt[3]{1+t^{2}}$, what is the position of the object at time $t=3$ ?
(A) 0.431
(B) 2.154
(C) 4.512
(D) 6.512
(E) 17.408
88. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area $S$ of a sphere with radius $r$ is $S=4 \pi r^{2}$ )
(A) $-108 \pi$
(B) $-72 \pi$
(C) $-48 \pi$
(D) $-24 \pi$
(E) $-16 \pi$
89. The function $f$ is continuous for $-2 \leq x \leq 2$ and $f(-2)=f(2)=0$. If there is no $c$, where $-2<c<2$, for which $f^{\prime}(c)=0$, which of the following statements must be true?
(A) For $-2<k<2, f^{\prime}(k)>0$.
(B) For $-2<k<2, f^{\prime}(k)<0$.
(C) For $-2<k<2, f^{\prime}(k)$ exists.
(D) For $-2<k<2, f^{\prime}(k)$ exists, but $f^{\prime}$ is not continuous.
(E) For some $k$, where $-2<k<2, f^{\prime}(k)$ does not exist.
90. The function $f$ is continuous on the closed interval [2,4] and twice differentiable on the open interval $(2,4)$. If $f^{\prime}(3)=2$ and $f^{\prime \prime}(x)<0$ on the open interval $(2,4)$, which of the following could be a table of values for $f$ ?
(A)

| $x$ | $f(x)$ |
| :---: | :---: |
| 2 | 2.5 |
| 3 | 5 |
| 4 | 6.5 |

(B)

| $x$ | $f(x)$ |
| :---: | :---: |
| 2 | 2.5 |
| 3 | 5 |
| 4 | 7 |

(C)

| $x$ | $f(x)$ |
| :---: | :---: |
| 2 | 3 |
| 3 | 5 |
| 4 | 6.5 |

(D)

| $x$ | $f(x)$ |
| :---: | :---: |
| 2 | 3 |
| 3 | 5 |
| 4 | 7 |

(E)

| $x$ | $f(x)$ |
| :---: | :---: |
| 2 | 3.5 |
| 3 | 5 |
| 4 | 7.5 |

91. What is the average value of $y=\frac{\cos x}{x^{2}+x+2}$ on the closed interval $[-1,3]$ ?
(A) -0.085
(B) 0.090
(C) 0.183
(D) 0.244
(E) 0.732

92. A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip $x$ miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?
(A) $\int_{0}^{4} f(x) d x$
(B) $7 \int_{0}^{4} f(x) d x$
(C) $28 \int_{0}^{4} f(x) d x$
(D) $\int_{0}^{7} f(x) d x$
(E) $4 \int_{0}^{7} f(x) d x$
