CALCULUS A/B

Unit 5—Skills Review

Name

Period: 2

Date:

These questions will approximate the questions you will have on the skills portion of chapter 5 Unit test

LT: I can find the derivative of a natural log function (5.1)

1) $f(x) = \ln(x^2 - 3x)$

$$\boxed{f'(x) = \frac{2x-3}{x^2-3x}}$$

2) $f(x) = \ln(\cos x)$

$$f'(x) = \frac{-\sin x}{\cos x}$$

$$\boxed{= -\tan(x)}$$

3) $f(x) = 4\ln(\sqrt{x^2 - 4})$

$$f(x) = 4\ln(x^2-4)^{\frac{1}{2}}$$

$$= 2\ln(x^2-4)$$

$$f'(x) = 2 \cdot \frac{2x}{x^2-4}$$

$$= \boxed{\frac{4x}{x^2-4}}$$

4) $f(x) = \ln(\frac{2x}{x^2+4})$

$$= \ln 2x - \ln x^2+4$$

$$= \frac{2}{2x} - \frac{2x}{x^2+4}$$

$$\boxed{= \frac{1}{x} - \frac{2x}{x^2+4}}$$

LT: I can find the equation of a tangent line (5.1)

Find the equation of the tangent line to the graph of *f* at the given point.

5) $f(x) = \ln(x^3 - 7)$; at x=2

$2, \ln(2^3-7)$

$2, \ln(1)$

$2, 0$

$f'(x) = \frac{3x^2}{x^3-7}$

at 2 = $\frac{12}{1}$

$$\boxed{y = 0 + 12(x-2)}$$

6) $f(x) = \frac{1}{2}x\ln(x^2)$; at x=1

$1, \frac{1}{2}(1)\ln(1)$

$1, 0$

$y = y_1 + m(x - x_1)$

$$\boxed{y = 0 + 1(x-1)}$$

$f'(x) = \frac{1}{2}\ln x^2 + \frac{1}{2}x \cdot \frac{1}{x^2} \cdot 2x$

$f'(x) = \ln x + 1$

$= \ln 1 + 1 = 1$

hmm.. crap question ;

7) $f(x) = $ ~~$\sin(x + \frac{\pi}{2})\ln(\cos x)$; at $x = \frac{\pi}{3}$~~

$\sin(x)\ln(\cos(x))$ at $x=0$

pt: $0, 0$

$y = 0 + 0(x-0)$

$f'(x) = \cos x \ln \cos x + \sin x \frac{1}{\cos x} \cdot -\sin x$

$f'(0) = \cos 0 \ln \cos 0 + \sin(0)\frac{1}{\cos 0} \cdot -\sin 0$

$1 \cdot \ln 1 + 0 \cdot \frac{1}{1} \cdot -0$

$f'(0) = 0$

LT: I can find the relative extrema and points of inflection (5.1)

crit pts and pts inflection when numerator is 0

a) locate any relative extrema

b) locate any piont of inflection

8) $f(x) = x\ln x$

a) $f'(x) = \ln x + x\frac{1}{x}$

$f'(x) = 0 = \ln x + 1$

$e^{-1} = e^{\ln x}$

$e^{-1} = x$

b) $f''(x) = \frac{1}{x}$

$f(x) = x\ln x$ only exists for $x > 0$

no point of inflection

x	e^{-2}	e^{-1}	1
$f'(x)$	-1	0	1

$x = e^{-1}$ is relative min

9) $f(x) = \frac{\ln x}{x}$

$1 - \ln x = 0$

$\ln x = 1$

$\boxed{x = e}$ relative max

$f'(x) = \frac{\frac{1}{x} \cdot x - \ln x}{x^2}$

$f'(x) = \frac{1 - \ln x}{x^2}$

x	$e^{1/2}$	e	e^2
$f'(x)$	$\frac{1}{2}$	0	-1

pt of inflection at $e^{3/2}$

since no values $x > 0$ will make denominator 0.

$f''(x) = \frac{-\frac{1}{x} \cdot x^2 - (1-\ln x)2x}{x^4} \Rightarrow \frac{-x - 2x + 2x\ln x}{x^4}$

$\frac{-3 + 2\ln x}{x^4} \Leftarrow \frac{-3x + 2x\ln x}{x^4}$

LT: I can integrate a rational function (5.2)

Find the integral for each:

10) $\frac{1}{6}\int \frac{6x}{3x^2-4}dx$

$u = 3x^2 - 4$

$du = 6x\,dx$

$$\boxed{\frac{1}{6}\ln|3x^2-4| + C}$$

11) $\int \frac{-3}{\tan x \cos^2 x}dx$

$\int \frac{-3}{\tan x}\left(\frac{1}{\cos^2 x}\right)dx$

$-3\int \frac{1}{\tan x}\sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$$\boxed{-3\ln|\tan x| + C}$$

12) $2\int \frac{\frac{1}{2}(6x^2-8x)}{x^3-2x^2-6}dx$

$u = x^3 - 2x^2 - 6$

$du = 3x^2 - 4x$

$$\boxed{2\ln|x^3-2x^2-6| + C}$$

13) $2\int \frac{\frac{1}{2}(4x+6)\cos(x^2+3x)}{\sin(x^2+3x)}dx$

$u = \sin(x^2+3x)$

$du = (2x+3)\cos(x^2+3x)dx$

$$\boxed{2\ln|\sin(x^2+3x)| + C}$$

~~$\int \frac{1}{u} du =$~~ $\ln|u|$

won't need for this test ($\int \sec x dx$) but will need $\int \sec^2 x\, dx$

LT: I can integrate a rational function when denominator is not the higher power (long division or u-sub) (5.2)

Find the integral for each:

14) $\int \frac{3x^2-5x+1}{x-2}dx$

$$x-2\overline{)3x^2-5x+1} \quad 3x+1$$
$$-(3x^2-6x)$$
$$x+1$$
$$-(x-2)$$
$$3$$

$\int 3x+1+\frac{3}{x-2}\,dx$

$\boxed{\frac{3x^2}{2}+x+3\ln|x-2|+c}$

15) $\int \frac{x^2-6x-20}{x+5}dx$

$$x+5\overline{)x^2-6x-20} \quad x-11$$
$$-(x^2+5x)$$
$$-11x-20$$
$$-(-11x-55)$$
$$35$$

$\int x-11+\frac{35}{x+5}\,dx$

$\boxed{\frac{x^2}{2}-11x+35\ln|x+5|+c}$

16) $-\frac{3}{2}\cdot\frac{3}{2}\int \frac{3\sin 2x}{\cos 2x}dx$

$u=\cos 2x$

$du=-2\sin 2x\,dx$

$\boxed{-\frac{3}{2}\ln|\cos 2x|+c}$

17) $\frac{1}{5}\int \sec(5x)5dx$

$=\int \sec x\,dx$

$u=5x$

$du=5dx$

$\frac{1}{5}\ln|\sec 5x+\tan 5x|+C$

LT: I can find the value of a definite integral of a rational function (5.2)

...keep answers in (natural log...ln) form.

18) $\int_3^6 \frac{2}{2x-5}dx$

$u=2x-5$

$du=2dx$

$\ln|2x-5|\Big|_3^6$

$\ln|7|-\ln 1=\boxed{\ln 7}$

19) $\frac{1}{2}\int_1^3 \frac{2(3x-1)}{3x^2-2x}dx$

$u=3x^2-2x$

$du=6x-2$

$\frac{1}{2}\ln|3x^2-2x|\Big|_1^3$

$\frac{1}{2}(\ln 21-\ln 1)=\boxed{\frac{1}{2}\ln 21}$

20) $\int_{\frac{\pi}{2}}^{2\pi} \sec x \sin x\, dx$

$-1\int_{\frac{\pi}{2}}^{2\pi}\left(\frac{1}{\cos x}\sin x\,dx\right)-1$

$u=\cos x$

$du=-\sin x\,dx$

$-\ln|\cos x|\Big|_{\frac{\pi}{2}}^{2\pi}$

$-\ln(\cos 2\pi)--\ln\cos\frac{\pi}{2}$

$-\ln(1)+\ln\cos\frac{\pi}{2}$

oops... no integral

21) $5\int_{\frac{\pi}{15}}^{\frac{\pi}{5}} \tan(5x)5dx$

$5\ln(\cos 5x)\Big|_{\frac{\pi}{15}}^{\frac{\pi}{5}}$

$5\ln(\cos\pi)-5\ln(\cos\frac{\pi}{3})$

$5\ln(0)-5\ln(\frac{1}{2})$

I goofed... ln(0) doesn't exist.

LT: I can find the equation of a function given the derivative and one point through the graph (5.2)

Find the integral for each:

22) $\frac{dy}{dx}=\frac{3}{2-x}$; (1,0)

$dy=\frac{3}{2-x}dx$

$\int dy=-\int-\frac{3}{2-x}dx$

$u=2-x$

$du=-dx$

$y=-\ln(2-x)+c$

$0=-\ln(2-1)+c$

$0=0+c \rightarrow c=0$

$y=-\ln|2-x|$

23) $\frac{dy}{dx}=\frac{x-2}{x}$; (-1,0)

$dy=\frac{x-2}{x}dx$

$\int dy=\int 1-\frac{2}{x}dx$

$y=x-2\ln x+c$

I goofed again! no $\ln(-1)$!

24) $\frac{dy}{dx}=\frac{sec^2 x}{\tan(x)+1}$; $(\pi,4)$

$\int dy=\int\frac{\sec^2 x}{\tan x+1}dx$

$y=\ln|\tan x+1|+c$

$4=\ln|\tan\pi+1|+c$

$4=\ln|0+1|+c$

$4=\ln|1|+c$

$4=0+c$

$4=c$

$\boxed{y=\ln|\tan x+1|+4}$

These (#25-28) will not be assessed on the chp. Test...though they will be on the AP exam in May.

There will be one bonus question on the M.C. test on Friday covering this material.

LT: I can find the inverse of a function and I understand their graphical relationship (reflection of each other across the y=x line) and other properties...such as $f^{-1}(f(x))=x$

LT: I can find the derivative of an inverse function

$$if\ f(x) is\ the\ original\ function, f^{-1}(x) is\ the\ inverse.$$

$$the\ derivative\ of\ the\ inverse\ \left[(f^{-1})'^{(x)}\right]=\frac{1}{f'(f^{-1}(x))}$$

Find the inverse function for both:

25) $y=e^{5x-2}$

$\ln x=\ln e^{5y-2}$

$\ln x=5y-2$

$\boxed{\frac{(\ln x)+2}{5}=y}$

26) $y=3+\ln(5x)$

$x=3+\ln(5y)$

$e^{x-3}=e^{\ln(5y)}$

$e^{x-3}=5y$

$\boxed{\frac{e^{x-3}}{5}=y}$

Find the derivative of each function's inverse at x=3

27) ~~$y=x^2+4x$~~

~~$f'(x)=2x+4$~~

~~$f^{-1}(x)\Rightarrow x=y^2+4y$~~

$(f^{-1})'(x)=\frac{1}{f'(f^{-1}(3))}$

$4e^{\frac{1}{4}\ln x}$ $4e^{\ln x}\left(f'\left(\frac{\ln 3}{4}\right)\right)$

28) $y=e^{4x}$

$y'=4e^{4x}$

$\boxed{4x}$

$y^{-1}\Rightarrow x=e^{4y}$

$\ln x=4y$

$\frac{\ln x}{4}=y^{-1}$

LT: I can differentiate an exponential function (with base *e)* (5.4)

Find the derivative for each:

29) $y = e^{4x-2}$

$y' = \boxed{4e^{4x-2}}$

30) $y = 6x^2e^{4x}$

p= p+p

$12xe^{4x} + 6x^2\cdot 4e^{4x}$

$\boxed{12xe^{4x} + 24x^2e^{4x}}$

31) $y = e^{x^2}\ln 2x$

$2xe^{x^2}\ln 2x + e^{x^2}\cdot 2\frac{1}{2x}$

$\boxed{2xe^{x^2}\ln 2x + \frac{e^{x^2}}{x}}$

32) $y = e^{\sin 4x}$

$e^{\sin 4x}\cdot\cos 4x\cdot 4$

$4e^{\sin 4x}\cos 4x$

LT: I can find the integral of an exponential function (with base *e)* (5.4)

33) $\frac{1}{3}\int e^{3x}3dx$

$u = 3x$
$du = 3dx$

$\boxed{\frac{1}{3}e^{3x}} + C$

34) $\frac{1}{3}\int 3x^2e^{x^3}dx$

$u = x^3$
$du = 3x^2$

$\boxed{\frac{1}{3}e^{x^3}} + C$

35) $\frac{1}{4}\int 4e^{4x}(e^{4x}-3)dx$

$u = e^{4x}-3$
$du = 4e^{4x}dx$

$\boxed{\frac{1}{4}(e^{4x}-3)} + C$

36) $\frac{1}{3}\int\frac{3e^{3x}}{e^{3x}+1}dx$

$u = e^{3x}+1$
$du = 3e^{3x}dx$

$\frac{1}{3}\int\frac{1}{u}du$

$\frac{1}{3}\ln(e^{3x}+1) + C$

LT: I can find the value of a definite integral of an exponential function (wit base e) (5.4)
...keep answers in terms of *e*

37) $\frac{1}{2}\int_0^3 e^{2x}2dx$

$\frac{1}{2}e^{2x}\Big|_0^3$

$\frac{1}{2}e^6 - \frac{1}{2}e^0$

$\boxed{\frac{1}{2}e^6 - \frac{1}{2}}$

38) $\int_0^{\frac{3\pi}{4}}\cos x\, e^{\sin x}dx$

$u = \sin x$
$du = \cos x\, dx$

$e^{\sin x}\Big|_0^{3\pi/4}$

$e^{\sin 3\pi/4} - e^0$

$\boxed{e^{\frac{\sqrt{2}}{2}} - 1}$

39) $\frac{1}{2}\int_0^2\frac{e^{2x}}{1+e^{2x}}2dx$

$u = 1+e^{2x}$
$du = e^{2x}\cdot 2dx$

LL: $x=0$ $u=2$

UL $x=2$ $u=1+e^4$

$\frac{1}{2}\int_2^{1+e^4}\frac{1}{u}du$

$\frac{1}{2}\ln(u)\Big|_2^{1+e^4}$

$\boxed{\frac{1}{2}\ln(1+e^4) - \frac{1}{2}\ln(2)}$

40) $5\int_1^4 e^{5x-3}\frac{1}{5}dx$

$5e^{5x-3}\Big|_1^4$

$\boxed{5e^{17} - 5e^2}$