

Derivatives, their meaning and finding equations of tangent lines:

Find the derivative for each

$$y = 2x^3 + 4x^2 + 5$$

Pg 157 #'s 9-25 odd

$$\frac{dy}{dx} = y' = 6x^2 + 8x$$

What is the slope of the tangent line when $x=2$.

$$y = 3x^2 + \frac{8}{x^2} + 5x \quad \text{slope at } x=2 \Rightarrow 6(2) + -\frac{16}{2^3} + 5$$

$$y' = 6x + -\frac{16}{x^3} + 5 \quad \Rightarrow 12 - 2 + 5$$

$\Rightarrow 15$ is the slope at $x=2$)

What is the equation of the tangent line to the curve $y = 2\cos x - \sin x$ at $x = \frac{\pi}{2}$

$\begin{cases} \text{need slope at } \frac{\pi}{2} \\ \text{need y-intercept at } x=\frac{\pi}{2} \end{cases}$

$$\begin{aligned} y &= 2\cos\frac{\pi}{2} - \sin\frac{\pi}{2} \\ y &= -1 \end{aligned}$$

$$\begin{aligned} \text{slope} \\ \frac{dy}{dx} &= -2\sin x - \cos x \\ &= -2\sin\frac{\pi}{2} - \cos\frac{\pi}{2} \\ &= -2(1) - 0 \\ &= -2 \end{aligned}$$

$$y = y_1 + m(x-x_1)$$

$$y = -1 + -2(x - \frac{\pi}{2})$$

Derivative Rules

What is the slope of the tangent line for each of the following:

Pg 157 #'s 29-43 odd

a) $y = 3x^2 \sin x$ at $x = \frac{\pi}{6}$

$$y' = 6x \sin x + 3x^2 \cos x$$

$$y' = 6 \cdot \frac{\pi}{6} \sin \frac{\pi}{6} + 3 \left(\frac{\pi}{6}\right)^2 \cos \frac{\pi}{6}$$

$$y' = \frac{\pi}{2} + \frac{3\pi^2}{36} \frac{\sqrt{2}}{2}$$

b) $y = \frac{x^2}{\sqrt[3]{5x-2}}$ at $x = 2$

$$y = \frac{x^2}{(5x-2)^{\frac{1}{3}}}$$

$$\begin{aligned} y' &= \frac{2x(5x-2)^{\frac{1}{3}} - x^2 \cdot \frac{1}{3}(5x-2)^{-\frac{2}{3}}(5)}{(5x-2)^{\frac{2}{3}}} \\ y' &= \frac{4(2) - 4 \cdot \frac{1}{3} \cdot \frac{1}{8} \cdot 5}{(8)^{\frac{2}{3}}} = \frac{8 - \frac{5}{3}}{4} = 1.583 \end{aligned}$$

c) $y = \sin(5x^2)$ at $x = -2$

$$y' = \cos(5x^2) \cdot 10x$$

$$y' = -20 \cos(20)$$

in degree mode

$$y' = -18.794$$

in radian mode

$$y' = -8.162$$

Implicit Differentiation:

Pg 157 #'s 77-81 odd

Find the derivative for:

$$5xy = x^2 + y^2$$

$$5y + 5x \frac{dy}{dx} = 2x + 2y \frac{dy}{dx}$$

$$-2y \frac{dy}{dx} + 5x \frac{dy}{dx} = 2x - 5y$$

$$\frac{dy}{dx} (-2y + 5x) = 2x - 5y$$

$$\frac{dy}{dx} = \frac{2x - 5y}{-2y + 5x}$$

$$\tan(x+y) = x^2$$

$$\sec^2(x+y) (1 + \frac{dy}{dx}) = 2x$$

$$1 + \frac{dy}{dx} = \frac{2x}{\sec^2(x+y)}$$

$$\frac{dy}{dx} = \frac{2x}{\sec^2(x+y)} - 1$$

Second Derivatives:

Pg 157 #'s 45-49 odd AND 71-75 odd

Find the second derivative of each:

$$y = x^3 + 5x^2 - 6x - 3$$

$$y' = 3x^2 + 10x - 6$$

$$y'' = 6x + 10$$

$$y = \sin(2x^3)$$

$$y' = 6x^2 \cos(2x^3)$$

$$y'' = 12x \cos(2x^3) + 6x^2 (-\sin(2x^3) \cdot 6x^2)$$

$$y' = 12x \cos(2x^3) + 36x^4 \sin(2x^3)$$

$$2xy = x^2 + 7x$$

$$2y + 2x \frac{dy}{dx} = 2x + 7$$

$$y' = \frac{dy}{dx} = \frac{2x - 2y + 7}{2x}$$

$$y'' = \frac{d^2y}{dx^2} = \frac{(2 - 2 \frac{dy}{dx})(2x) - (2x - 2y + 7)(2)}{4x^2}$$

$$= \frac{4x - 4x \frac{dy}{dx} - 4x + 4y - 14}{4x^2} = \frac{4x - 4x \left(\frac{2x - 2y + 7}{2x}\right) - 4x + 4y - 14}{4x^2}$$

Showing Differentiability:

Show that function f is differentiable at $x=2$

Check Scubamoose.weebly.com for examples

$$f(x) = \begin{cases} x^2 + 2x & ; \quad x \leq 2 \\ 6x - 4 & ; \quad x > 2 \end{cases}$$

continues at x=2

$$\begin{aligned} \textcircled{1} \quad f(2) &= 2^2 + 2(2) = 8 \\ \textcircled{2} \quad \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^-} f(x) \\ 12 - 4 &= 8^2 + 2(2) \\ \cancel{8} &= \cancel{8} + 2 \end{aligned}$$

If $f(x)$ is differentiable at $x=3$, what is the value of $a+b$?

$$f(x) = \begin{cases} ax^2 + 4x & ; \quad x \leq 2 \\ bx - 8 & ; \quad x > 2 \end{cases}$$

Conclusions

$$\text{at } x=2 \quad ax^2 + 4x = 6x - 8$$

$$4a + 8 = 2b - 8$$

$$4a + 16 = 2b$$

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(4) derivatives same from negative and positive direction

from negative side of 2 derivative is $at x=2 \rightarrow 2x+2$	from positive side at 2 the derivative is 6 6
$y = x^2 + 2$	$6 = 6$
\therefore slope of tangent same from either direction	

Related Rates

Pg 160 #'s 9-11 odd

The volume of a cylinder with radius r and height h is given by $V = \pi r^2 h$. The radius and height of the cylinder are increasing at constant rates. The radius is expanding at $\frac{1\text{cm}}{3\text{sec}}$ and the height at $\frac{1\text{cm}}{2\text{sec}}$. At what rate, in *cubic cm per second*, is the volume of the cylinder increasing when the cylinder's height is 9cm and the radius is 4 cm?

$$V = \pi r^2 \cdot h$$

$$\frac{dv}{dt} = 2\pi rh \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = 2\pi (4h)(a) \cdot \frac{1}{3\pi a^2} + \pi (4h)^2 \cdot \frac{1}{2\pi a}$$

$$\frac{dV}{dt} = 24\pi \frac{m^3}{sec} + 8\pi \frac{m^3}{sec}$$

$$\frac{dV}{dt} = 32\pi \frac{m^3}{sec}$$

$$\frac{dr}{dt} = \frac{1}{3 \text{ sec}} \quad \frac{dh}{dt} = \frac{1}{2 \text{ sec}}$$

$$b=9 \quad r=4$$

Other examples.

1) $f(x) = (x^3 - 5)$

a) Write an equation of the tangent to the graph of f at $x = 3$

$$f'(x) = 3x^2 - 5$$

$$f'(x) = 22$$

$$Y = 22 + 22(x-3)$$

(3, 22)

($3^3 - 5$)

b) Find the values of x for which the graph of f has a horizontal tangent line.

$$3x^2 - 5 = 0$$

$$3x^2 = 5$$

$$x^2 = \frac{5}{3}$$

c) Find $f''(x)$

$$f'(x) = 3x^2 - 5$$

$$f''(x) = 6x$$

$$x = \pm \sqrt{\frac{5}{3}} \text{ or } \text{approx } 1.290 \text{ or } -1.290$$

2) Given this table

x	f(x)	f'(x)	g(x)	g'(x)
2	-3	1	5	-2
5	4	7	-1	2

a) If $h(x) = \frac{f(x)}{g(x)}$, find $h'(2)$

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$h'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{(g(2))^2} \Rightarrow \frac{(1)(5) - (-3)(-1)}{(5)^2} = \frac{5 - 6}{25} = -\frac{1}{25}$$

b) If $j(x) = f(g(x))$, find $j'(2)$

$$\boxed{-14}$$

c) If $k(x) = \sqrt{f(x)}$, find $k'(5)$

$$k(x) = (f(x))^{\frac{1}{2}}$$

$$k'(x) = \frac{1}{2}(f(x))^{-\frac{1}{2}} \cdot f'(x)$$

$$k'(5) = \frac{1}{2}(f(5))^{-\frac{1}{2}} \cdot f'(5)$$

$$k'(5) = \frac{1}{2}(4)^{-\frac{1}{2}} \cdot 7$$

$$= \frac{1}{2} \left(\frac{1}{2}\right) \cdot 7$$

$$= \boxed{\frac{7}{4}}$$

$$j(x) = f'(g(x)) \cdot g'(x)$$

$$j(2) = f'(g(2)) \cdot g'(2)$$

$$= f'(5) \cdot (-2)$$

$$= (7) \cdot (-2)$$

$$= -14$$