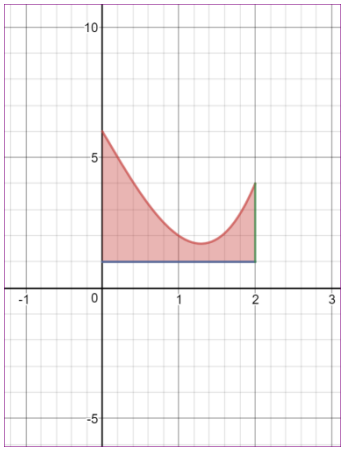
This FRQ question is equally likely to be a non-calculator as it is a calculator question

AREA BETWEEN 2 CURVES

The shaded region is enclosed by the y-axis, the vertical line $x=2$ and the graphs of $f(x)=x^3-5x+6$ and the horizontal line $y=1$.

a) Find the area of the shaded region



Steps (other methods work…this is just what your teacher advises)

1. Identify which is the function 'higher' or 'above' the other
2. Determine the 'interval' (integration limits) needed
3. Find the volume under the 'above' function (integral)
4. Find the volume under the 'below' function (integral)
5. Subtract the volume of the 'below' function from the 'above' function

Challenges:

1:

2:

3*:

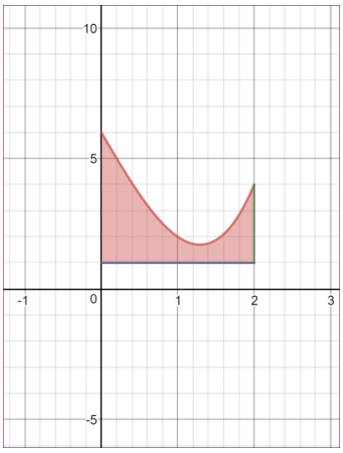
Calculator notes and additional steps that likely will be required

Area of a solid Revolved around an axis

The shaded region is enclosed by the y-axis, the vertical line $x=2$ and the graphs of $f(x)=x^3-5x+6$ and the horizontal line $y=1$.

First Type--axis is above (complete as a non-calculator example)

Find the volume of the solid generated when the shaded region is rotated about the horizontal line y=8.



Steps

When a solid is generated by rotating a figure around an axis
*video

Steps to find a volume of the solid:

1: Sketch/draw in the axis of rotation (what line are you rotating about?)

2: We will be determining the volume of 2 solids

1):

2):

3: Identify which is the function further from the axis of rotation (the **outer curve**).

a. This will be the 'block of wood' you are staring with before you 'carve out' the middle

b. Notice how the radius changes as you move along the x-axis.

Write the radius as a function of this outer curve

Use this radius in the integral to determine the volume of the solid as a sum of an **infinite number of cylinders**, each ***infinitely thin***.

4: Identify the function which is closer to the axis of rotation (this should be easy since you already identified the outer chunk ☺! (this will be the inner curve).

a. This will be the stuff you are to 'carve out' of our initial block of wood

b. b. Notice how this radius changes as you move along the x-axis.

Write the radius as a function of this outer curve

Use this radius in the integral to determine the volume of the solid as a sum of an **infinite number of cylinders**, each ***infinitely thin***.

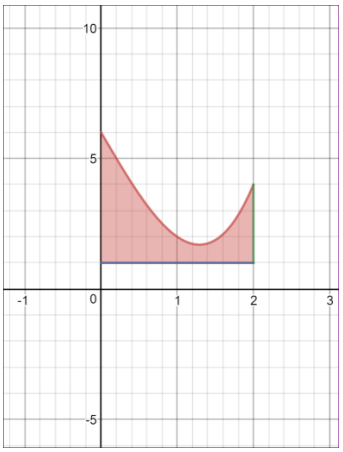
5: Subtract the 'carved out' volume from the 'initial chunk' volume.

Area of a solid Revolved around an axis … cont'd

Second Type--axis is below (complete as a non-calculator example)

The shaded region is enclosed by the y-axis, the vertical line $x=2$ and the graphs of $f(x)=x^3-5x+6$ and the horizontal line $y=1$.

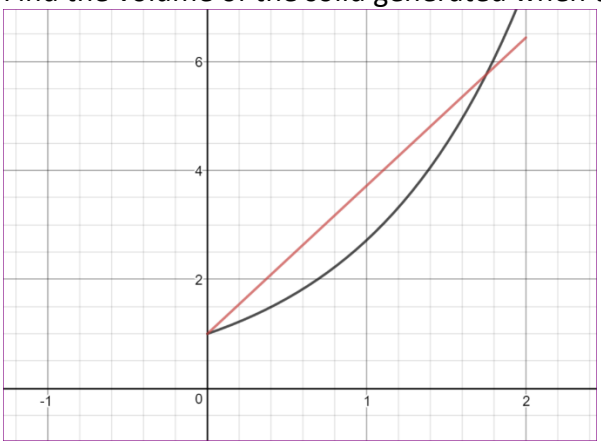
Find the volume of the solid generated when the shaded region is rotated about the horizontal line y= -1



Third Type--axis of rotation is the y-axis (versus a horizontal line) … (complete as a calculator example)

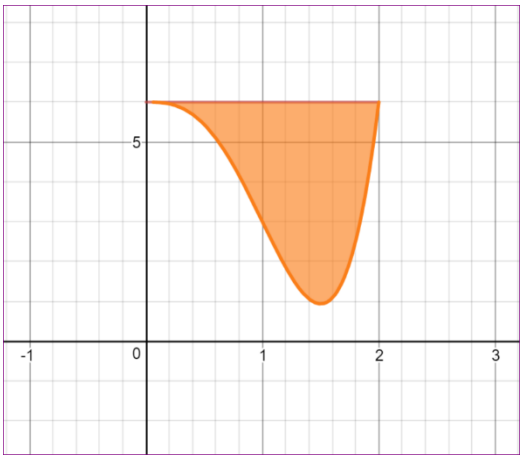
The shaded region is enclosed by the graphs of $f(x)=e^x$ and the horizontal line $y=ex+1$.

Find the volume of the solid generated when the shaded region is rotated about the y-axis



Areas of Volumes with known cross sections

The shaded region is enclosed by the graphs of $f(x)=3x^4-6x^3+6$ and the horizontal line $y=6$.



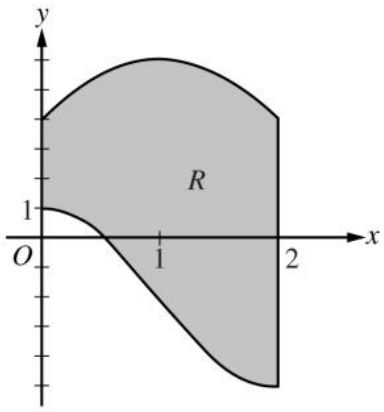
a) The shaded region is the base of a solid. For this solid, each cross section perpendicular to the x-axis is **a square**. Find the volume of the solid.

b) The shaded region is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are **semi-circles**. Find the volume of the solid.

c) The shaded region is the base of a solid. For this solid, the cross sections perpendicular to the x-axis is an **isosceles right triangle** with a leg in the shaded region. Find the volume of the solid.

d) The shaded region is the base of a solid. For this solid, the cross sections perpendicular to the x-axis is a **rectangle** whose height is 4 times the length of the base in the shaded region. Find the volume of the solid

2019 Question #5 (No Calculator)



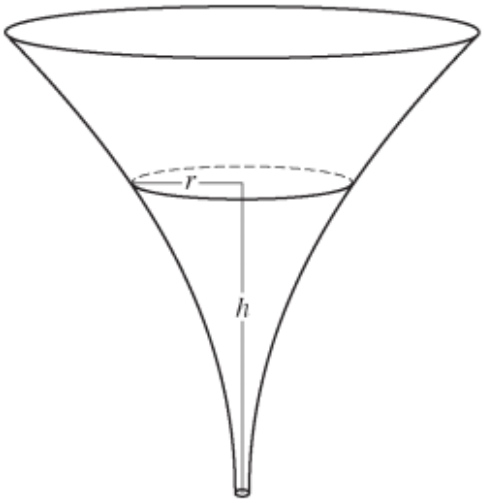
5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure above.

   (a) Find the area of R.

(b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \frac{1}{x+3}$. Find the volume of the solid.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

2016 Question #5 (No Calculator)



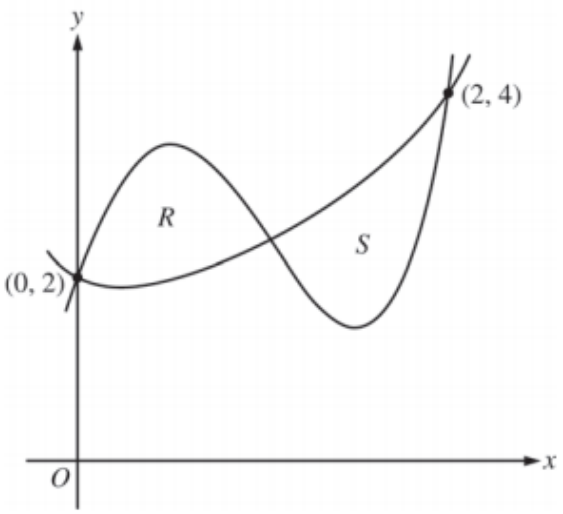
5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

2015 Question #2 (Calculator OK)



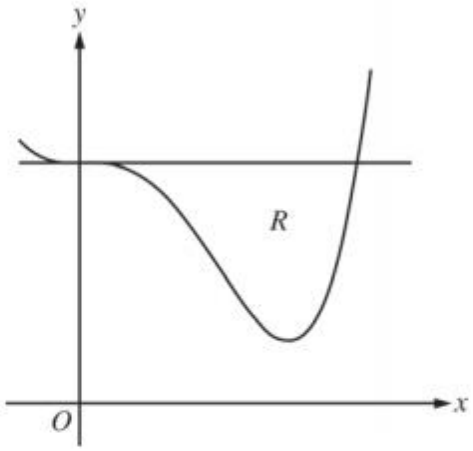
2. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

   (a) Find the sum of the areas of regions R and S.

(b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

2014 Question #2 (Calculator OK)



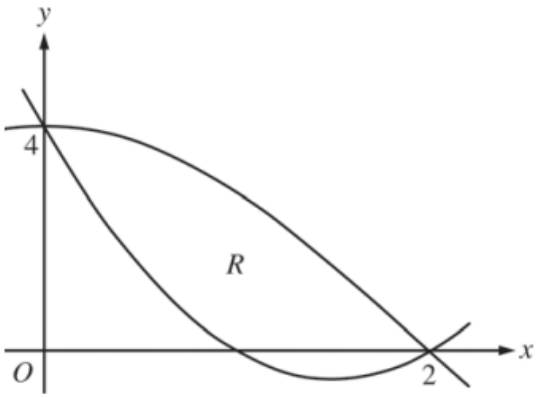
2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

   (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

2013 Question #5 (No Calculator)



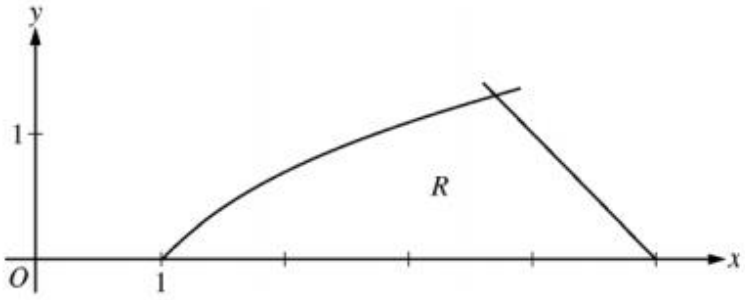
5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

2012 Question #2 (Calculator OK)



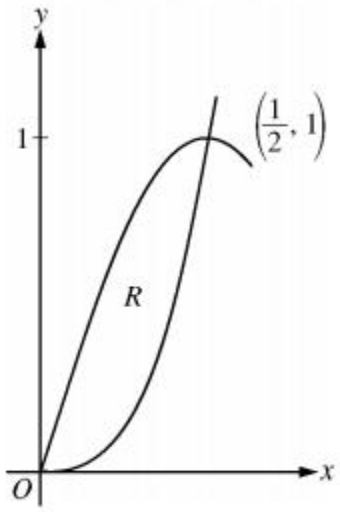
2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

   (a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

2011 Question #3



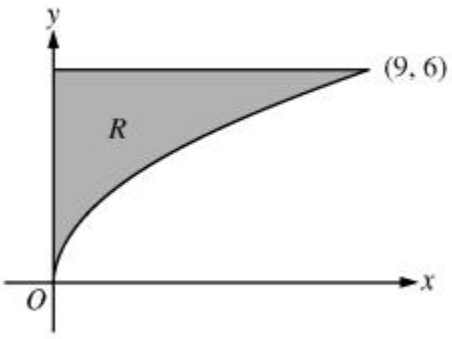
3. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

Question 4



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.