

Polar Functions

FRQ's

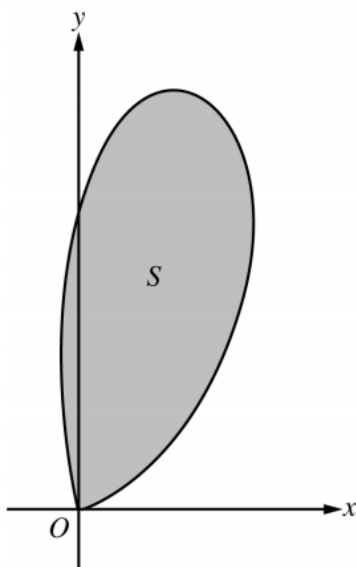
2019

2018

2017

2014

2013



2. Let S be the region bounded by the graph of the polar curve $r(\theta) = 3\sqrt{\theta} \sin(\theta^2)$ for $0 \leq \theta \leq \sqrt{\pi}$, as shown in the figure above.

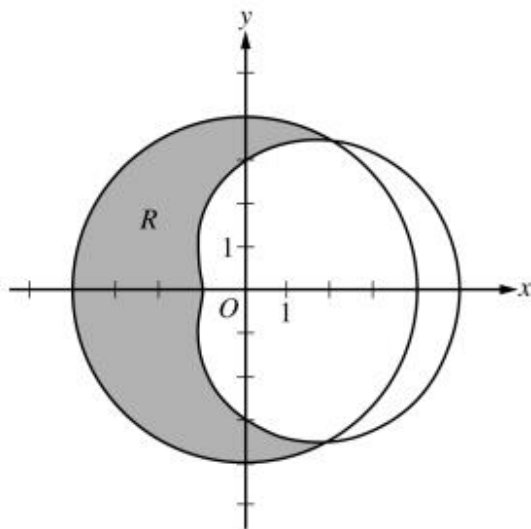
(a) Find the area of S .

- (b) What is the average distance from the origin to a point on the polar curve $r(\theta) = 3\sqrt{\theta} \sin(\theta^2)$ for $0 \leq \theta \leq \sqrt{\pi}$?

- (c) There is a line through the origin with positive slope m that divides the region S into two regions with equal areas. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of m .

- (d) For $k > 0$, let $A(k)$ be the area of the portion of region S that is also inside the circle $r = k \cos \theta$. Find

$$\lim_{k \rightarrow \infty} A(k).$$

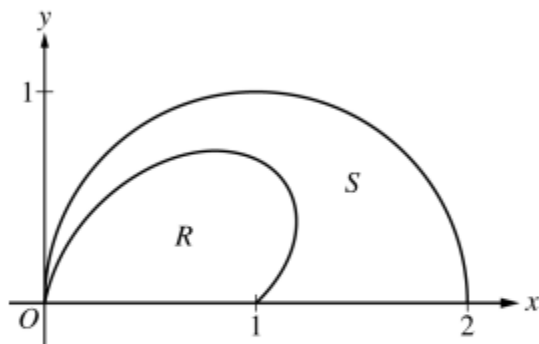
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5. The graphs of the polar curves $r = 4$ and $r = 3 + 2 \cos \theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.
- (a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2 \cos \theta$, as shown in the figure above. Write an expression involving an integral for the area of R .

(b) Find the slope of the line tangent to the graph of $r = 3 + 2 \cos \theta$ at $\theta = \frac{\pi}{2}$.

(c) A particle moves along the portion of the curve $r = 3 + 2 \cos \theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

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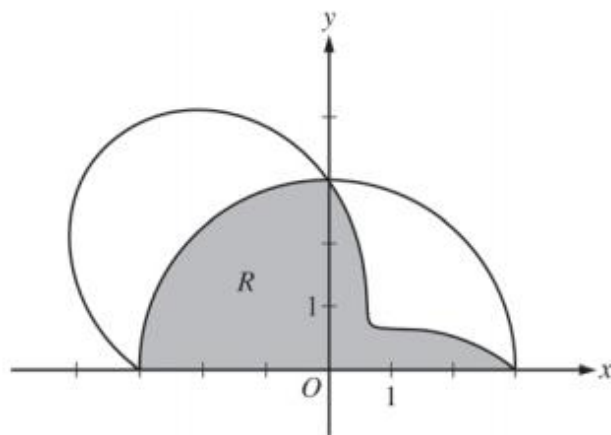
2. The figure above shows the polar curves $r = f(\theta) = 1 + \sin \theta \cos(2\theta)$ and $r = g(\theta) = 2 \cos \theta$ for $0 \leq \theta \leq \frac{\pi}{2}$. Let R be the region in the first quadrant bounded by the curve $r = f(\theta)$ and the x -axis. Let S be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the x -axis.

(a) Find the area of R .

- (b) The ray $\theta = k$, where $0 < k < \frac{\pi}{2}$, divides S into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .

- (c) For each θ , $0 \leq \theta \leq \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find w_A , the average value of $w(\theta)$ over the interval $0 \leq \theta \leq \frac{\pi}{2}$.

- (d) Using the information from part (c), find the value of θ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of θ ? Give a reason for your answer.

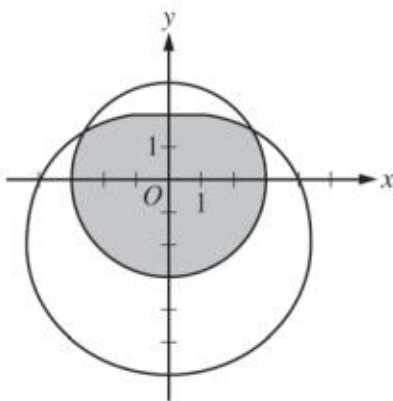
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2. The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \leq \theta \leq \pi$.
- (a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R .
- (b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

- (c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

- (d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

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2. The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin \theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.
- (a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin \theta$. Find the area of S .

- (b) A particle moves along the polar curve $r = 4 - 2\sin \theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \leq t \leq 2$ for which the x -coordinate of the particle's position is -1 .
- (c) For the particle described in part (b), find the position vector in terms of t . Find the velocity vector at time $t = 1.5$.