Non Calc 2007

4. A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

(a) Find the time t at which the particle is farthest to the left. Justify your answer.

(b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

a) find absolute minimums (compare crit points to end points) lowest value

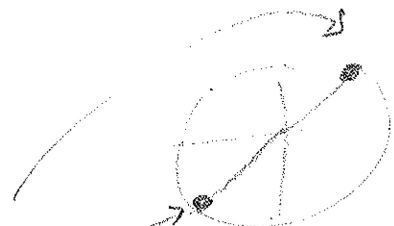
$$x(t) = e^{-t}\sin t$$

$$y'(t) = -e^{-t}\sin t + e^{-t}\cos t$$

$y'(t)$ is crit pts

$$0 = -e^{-t}\sin t + e^{-t}\cos t$$
$$+e^{-t}\sin t \qquad +e^{-t}\sin t$$

$$\frac{e^{-t}\sin t}{e^{-t}} = \frac{e^{-t}\cos t}{e^{-t}}$$



$\sin t = \cos t$ at $t = \frac{\pi}{4}, \frac{5\pi}{4}$

compare

x	0	$\frac{\pi}{4}$	$\frac{5\pi}{4}$	2π
y	0	pos	neg	0

→ plug into $x(t) = e^{-t}\sin t$

ex $x(0) = e^{-0}\sin 0 = 0$

$e^{-\frac{5\pi}{4}} \sin\frac{5\pi}{4}$

(some positive value) $\left(-\frac{\sqrt{2}}{2}\right)$

neg number

when the time is $\frac{5\pi}{4}$, the position will be at it's most left!

$$x(t) = e^{-t}\sin t$$

b) $$x'(t) = -e^{-t}\sin t + e^{-t}\cos t$$

$$x''(t) = \left(e^{-t}\sin t + -e^{-t}\cos t\right) + \left(-e^{-t}\cos t + e^{-t}(-\sin t)\right)$$

$$\cancel{e^{-t}\sin t} - e^{-t}\cos t - e^{-t}\cos t \cancel{- e^{-t}\sin t}$$

$$x''(t) = -2e^{-t}\cos t$$

$$A(x''(t)) + x'(t) + x(t) = 0$$

$$A(-2e^{-t}\cos t) + (-e^{-t}\sin t + e^{-t}\cos t) + e^{-t}\sin t = 0$$

$$A(-2e^{-t}\cos t) \cancel{-e^{-t}\sin t} + e^{-t}\cos t \cancel{+e^{-t}\sin t} = 0$$

$$A(-2e^{-t}\cos t) + e^{-t}\cos t = 0$$

$$\boxed{A = \frac{1}{2}}$$

Non Calc 2008

6. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

(c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

(d) Find $\lim_{x \to 0^+} f(x)$.

a) tangent line $\Rightarrow$ need slope $(f'(e^2))$
and 1 point $(e^2, f(e^2))$

point $f(e^2) = \frac{\ln e^2}{e^2} = \frac{2}{e^2}$

$f'(e^2) = \frac{1 - \ln e^2}{(e^2)^2} = \frac{1-2}{e^4} = \frac{-1}{e^4}$

$y = y_1 + m(x - x_1)$

$\boxed{y = \frac{2}{e^2} - \frac{1}{e^4}(x - e^2)}$ or same algebraic equivalent.

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b) $f'(x) = \frac{1 - \ln x}{x^2}$ since $x > 0$ need to find values make numerator 0. i.e. $\rightarrow$ $1 - \ln x = 0$

$-\ln x = -1$

$\ln x = 1$

e e

$\boxed{x = e}$

$1 - \ln x = 0$ when $x = e$

$1 - \ln x < 0$ when $x > e$ ($\ln x > 1$ when $x > 0$)

$1 - \ln x > 0$ when $x < e$ ($\ln x < 1$ when $x < 0$)

	$x < e$	$x = e$	$x > e$
$f'(x) \rightarrow$	pos	0	neg

$\boxed{x = e \text{ is a relative maximum}}$

c) $f(x) = \frac{\ln x}{x}$

$f'(x) = \frac{1-\ln x}{x^2}$

$f''(x) = \frac{-\frac{1}{x}(x^2) - (1-\ln x)(2x)}{x^4}$

$x > 0$

$= \frac{-x - (2x - 2x\ln x)}{x^4}$

$= \frac{-x - 2x + 2x\ln x}{x^4}$

$= \frac{-3x + 2x\ln x}{x^4}$

$f''(x) = \frac{-3 + 2\ln x}{x^3}$

$f''(x) = 0$ when numerator is zero

$-3 + 2\ln x = 0$

$2\ln x = 3$

$e^{\ln x} = e^{\frac{3}{2}}$

$\boxed{x = e^{3/2} \text{ is the only possible point of inflection}}$

d) $\lim_{x \to 0^+} f(x) \Rightarrow \lim_{x \to 0^+} \frac{\ln(x)}{x}$

Note
(can't approach from negative since $\ln(\text{negative})$ doesn't exist)

as $x \to 0^+$

- the numerator will become an increasingly large negative number
- the denominator will become an increasingly small positive number,
- the ratio will be an increasingly large negative number

or

$$\boxed{\lim_{x \to 0^+} f(x) = -\infty}$$

this was unnecessary for the problem... only written for student ~~bene~~ benefit.