

2018 Question 2

2. An environmental science teacher at a high school with a large population of students wanted to estimate the proportion of students at the school who regularly recycle plastic bottles. The teacher selected a random sample of students at the school to survey. Each selected student went into the teacher's office, one at a time, and was asked to respond yes or no to the following question.

Do you regularly recycle plastic bottles?

Based on the responses, a 95 percent confidence interval for the proportion of all students at the school who would respond yes to the question was calculated as  $(0.584, 0.816)$ .

- (a) How many students were in the sample selected by the environmental science teacher?
- (b) Given the method used by the environmental science teacher to collect the responses, explain how bias might have been introduced and describe how the bias might affect the point estimate of the proportion of all students at the school who would respond yes to the question.

(c) The statistics teacher at the high school was concerned about the potential bias in the survey. To obtain a potentially less biased estimate of the proportion, the statistics teacher used an alternate method for collecting student responses. A random sample of 300 students was selected, and each student was given the following instructions on how to respond to the question.

- In private, flip a fair coin.
- If heads, you must respond no, regardless of whether you regularly recycle.
- If tails, please truthfully respond yes or no.

(i) What is the expected number of students from the sample of 300 who would be required to respond no because the coin flip resulted in heads?

(ii) The results of the sample showed that 213 of the 300 selected students responded no. Based on the results of the sample, give a point estimate for the proportion of all students at the high school who would respond yes to the question.

2017 Question #2

2. The manager of a local fast-food restaurant is concerned about customers who ask for a water cup when placing an order but fill the cup with a soft drink from the beverage fountain instead of filling the cup with water. The manager selected a random sample of 80 customers who asked for a water cup when placing an order and found that 23 of those customers filled the cup with a soft drink from the beverage fountain.
  - (a) Construct and interpret a 95 percent confidence interval for the proportion of all customers who, having asked for a water cup when placing an order, will fill the cup with a soft drink from the beverage fountain.

- (b) The manager estimates that each customer who asks for a water cup but fills it with a soft drink costs the restaurant \$0.25. Suppose that in the month of June 3,000 customers ask for a water cup when placing an order. Use the confidence interval constructed in part (a) to give an interval estimate for the cost to the restaurant for the month of June from the customers who ask for a water cup but fill the cup with a soft drink.

5. A polling agency showed the following two statements to a random sample of 1,048 adults in the United States.

Environment statement: Protection of the environment should be given priority over economic growth.

Economy statement: Economic growth should be given priority over protection of the environment.

The order in which the statements were shown was randomly selected for each person in the sample. After reading the statements, each person was asked to choose the statement that was most consistent with his or her opinion. The results are shown in the table.

	Environment Statement	Economy Statement	No Preference
Percent of sample	58%	37%	5%

- (a) Assume the conditions for inference have been met. Construct and interpret a 95 percent confidence interval for the proportion of all adults in the United States who would have chosen the economy statement.
- (b) One of the conditions for inference that was met is that the number who chose the economy statement and the number who did not choose the economy statement are both greater than 10. Explain why it is necessary to satisfy that condition.

- (c) A suggestion was made to use a two-sample  $z$ -interval for a difference between proportions to investigate whether the difference in proportions between adults in the United States who would have chosen the environment statement and adults in the United States who would have chosen the economy statement is statistically significant. Is the two-sample  $z$ -interval for a difference between proportions an appropriate procedure to investigate the difference? Justify your answer.

2015 Question #2

2. To increase business, the owner of a restaurant is running a promotion in which a customer's bill can be randomly selected to receive a discount. When a customer's bill is printed, a program in the cash register randomly determines whether the customer will receive a discount on the bill. The program was written to generate a discount with a probability of 0.2, that is, giving 20 percent of the bills a discount in the long run. However, the owner is concerned that the program has a mistake that results in the program not generating the intended long-run proportion of 0.2.

The owner selected a random sample of bills and found that only 15 percent of them received discounts. A confidence interval for  $p$ , the proportion of bills that will receive a discount in the long run, is  $0.15 \pm 0.06$ . All conditions for inference were met.

(a) Consider the confidence interval  $0.15 \pm 0.06$ .

- (i) Does the confidence interval provide convincing statistical evidence that the program is not working as intended? Justify your answer.

- (ii) Does the confidence interval provide convincing statistical evidence that the program generates the discount with a probability of 0.2 ? Justify your answer.

A second random sample of bills was taken that was four times the size of the original sample. In the second sample 15 percent of the bills received the discount.

- (b) Determine the value of the margin of error based on the second sample of bills that would be used to compute an interval for  $p$  with the same confidence level as that of the original interval.

- (c) Based on the margin of error in part (b) that was obtained from the second sample, what do you conclude about whether the program is working as intended? Justify your answer.

2014 Question #3

3. Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.
- (a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?
- (b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.

Part c is from unit 5, but give it a go during unit 3 anyhow!

- (c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?

**Questions 1-5****Spend about 65 minutes on this part of the exam.****Percent of Section II score—75**

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. An environmental group conducted a study to determine whether crows in a certain region were ingesting food containing unhealthy levels of lead. A biologist classified lead levels greater than 6.0 parts per million (ppm) as unhealthy. The lead levels of a random sample of 23 crows in the region were measured and recorded. The data are shown in the stemplot below.

**Lead Levels**

2		8
3		0
3		5 8 8
4		1 1 2
4		6 8 8
5		0 1 2 2 3 4
5		9 9
6		3 4
6		6 8

Key: 2|8 = 2.8 ppm

- (a) What proportion of crows in the sample had lead levels that are classified by the biologist as unhealthy?

- (b) The mean lead level of the 23 crows in the sample was 4.90 ppm and the standard deviation was 1.12 ppm. Construct and interpret a 95 percent confidence interval for the mean lead level of crows in the region.

2010 Question #2

2. A local radio station plays 40 rock-and-roll songs during each 4-hour show. The program director at the station needs to know the total amount of airtime for the 40 songs so that time can also be programmed during the show for news and advertisements. The distribution of the lengths of rock-and-roll songs, in minutes, is roughly symmetric with a mean length of 3.9 minutes and a standard deviation of 1.1 minutes.
- (a) Describe the sampling distribution of the sample mean song lengths for random samples of 40 rock-and-roll songs.

- (b) If the program manager schedules 80 minutes of news and advertisements for the 4-hour (240-minute) show, only 160 minutes are available for music. Approximately what is the probability that the total amount of time needed to play 40 randomly selected rock-and-roll songs exceeds the available airtime?

2010 Question #3

3. A humane society wanted to estimate with 95 percent confidence the proportion of households in its county that own at least one dog.
  - (a) Interpret the 95 percent confidence level in this context.

The humane society selected a random sample of households in its county and used the sample to estimate the proportion of all households that own at least one dog. The conditions for calculating a 95 percent confidence interval for the proportion of households in this county that own at least one dog were checked and verified, and the resulting confidence interval was  $0.417 \pm 0.119$ .

- (b) A national pet products association claimed that 39 percent of all American households owned at least one dog. Does the humane society's interval estimate provide evidence that the proportion of dog owners in its county is different from the claimed national proportion? Explain.

- (c) How many households were selected in the humane society's sample? Show how you obtained your answer.