2019 Question #3

3. A medical researcher surveyed a large group of men and women about whether they take medicine as prescribed. The responses were categorized as never, sometimes, or always. The relative frequency of each category is shown in the table.

| | Never | Sometimes | Always | Total |
|---|---|---|---|---|
| Men | 0.0564 | 0.2016 | 0.2120 | 0.4700 |
| Women | 0.0636 | 0.1384 | 0.3280 | 0.5300 |
| Total | 0.1200 | 0.3400 | 0.5400 | 1.0000 |

(a) One person from those surveyed will be selected at random.

(i) What is the probability that the person selected will be someone whose response is never and who is a woman?

(ii) What is the probability that the person selected will be someone whose response is never or who is a woman?

(iii) What is the probability that the person selected will be someone whose response is never given that the person is a woman?

(b) For the people surveyed, are the events of being a person whose response is never and being a woman independent? Justify your answer.

(c) Assume that, in a large population, the probability that a person will always take medicine as prescribed is 0.54. If 5 people are selected at random from the population, what is the probability that at least 4 of the people selected will always take medicine as prescribed? Support your answer.

2019 Question #5

5. A company that manufactures smartphones developed a new battery that has a longer life span than that of a traditional battery. From the date of purchase of a smartphone, the distribution of the life span of the new battery is approximately normal with mean 30 months and standard deviation 8 months. For the price of $50, the company offers a two-year warranty on the new battery for customers who purchase a smartphone. The warranty guarantees that the smartphone will be replaced at no cost to the customer if the battery no longer works within 24 months from the date of purchase.

   (a) In how many months from the date of purchase is it expected that 25 percent of the batteries will no longer work? Justify your answer.

(b) Suppose one customer who purchases the warranty is selected at random. What is the probability that the customer selected will require a replacement within 24 months from the date of purchase because the battery no longer works?

(c) The company has a gain of \$50 for each customer who purchases a warranty but does not require a replacement. The company has a loss (negative gain) of \$150 for each customer who purchases a warranty and does require a replacement. What is the expected value of the gain for the company for each warranty purchased?

2018 Question #3

3. Approximately 3.5 percent of all children born in a certain region are from multiple births (that is, twins, triplets, etc.). Of the children born in the region who are from multiple births, 22 percent are left-handed. Of the children born in the region who are from single births, 11 percent are left-handed.

(a) What is the probability that a randomly selected child born in the region is left-handed?

(b) What is the probability that a randomly selected child born in the region is a child from a multiple birth, given that the child selected is left-handed?

(c) A random sample of 20 children born in the region will be selected. What is the probability that the sample will have at least 3 children who are left-handed?

2017 Question #3

3. A grocery store purchases melons from two distributors, J and K. Distributor J provides melons from organic farms. The distribution of the diameters of the melons from Distributor J is approximately normal with mean 133 millimeters (mm) and standard deviation 5 mm.

   (a) For a melon selected at random from Distributor J, what is the probability that the melon will have a diameter greater than 137 mm?

Distributor K provides melons from nonorganic farms. The probability is 0.8413 that a melon selected at random from Distributor K will have a diameter greater than 137 mm. For all the melons at the grocery store, 70 percent of the melons are provided by Distributor J and 30 percent are provided by Distributor K.

(b) For a melon selected at random from the grocery store, what is the probability that the melon will have a diameter greater than 137 mm?

(c) Given that a melon selected at random from the grocery store has a diameter greater than 137 mm, what is the probability that the melon will be from Distributor J?

2016 Question #4

4. A company manufactures model rockets that require igniters to launch. Once an igniter is used to launch a rocket, the igniter cannot be reused. Sometimes an igniter fails to operate correctly, and the rocket does not launch. The company estimates that the overall failure rate, defined as the percent of all igniters that fail to operate correctly, is 15 percent.

   A company engineer develops a new igniter, called the super igniter, with the intent of lowering the failure rate. To test the performance of the super igniters, the engineer uses the following process.

   Step 1: One super igniter is selected at random and used in a rocket.

   Step 2: If the rocket launches, another super igniter is selected at random and used in a rocket.

   Step 2 is repeated until the process stops. The process stops when a super igniter fails to operate correctly or 32 super igniters have successfully launched rockets, whichever comes first. Assume that super igniter failures are independent.

   (a) If the failure rate of the super igniters is 15 percent, what is the probability that the first 30 super igniters selected using the testing process successfully launch rockets?

(b) Given that the first 30 super igniters successfully launch rockets, what is the probability that the first failure occurs on the thirty-first or the thirty-second super igniter tested if the failure rate of the super igniters is 15 percent?

(c) Given that the first 30 super igniters successfully launch rockets, is it reasonable to believe that the failure rate of the super igniters is less than 15 percent? Explain.

2015 Question #3

3. A shopping mall has three automated teller machines (ATMs). Because the machines receive heavy use, they sometimes stop working and need to be repaired. Let the random variable $X$ represent the number of ATMs that are working when the mall opens on a randomly selected day. The table shows the probability distribution of $X$.

| Number of ATMs working when the mall opens | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Probability | 0.15 | 0.21 | 0.40 | 0.24 |

(a) What is the probability that at least one ATM is working when the mall opens?

(b) What is the expected value of the number of ATMs that are working when the mall opens?

(c) What is the probability that all three ATMs are working when the mall opens, given that at least one ATM is working?

(d) Given that at least one ATM is working when the mall opens, would the expected value of the number of ATMs that are working be less than, equal to, or greater than the expected value from part (b) ? Explain.

2014 Question #2

2. Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.

   (a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

   (b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

(c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

> Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

2013 Question #3

3. Each full carton of Grade A eggs consists of 1 randomly selected empty cardboard container and 12 randomly selected eggs. The weights of such full cartons are approximately normally distributed with a mean of 840 grams and a standard deviation of 7.9 grams.

   (a) What is the probability that a randomly selected full carton of Grade A eggs will weigh more than 850 grams?

(b) The weights of the empty cardboard containers have a mean of 20 grams and a standard deviation of 1.7 grams. It is reasonable to assume independence between the weights of the empty cardboard containers and the weights of the eggs. It is also reasonable to assume independence among the weights of the 12 eggs that are randomly selected for a full carton.

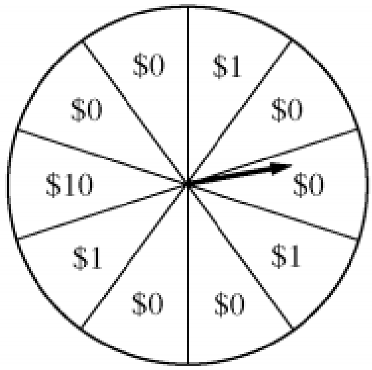
Let the random variable $X$ be the weight of a single randomly selected Grade A egg.

i) What is the mean of $X$ ?

ii) What is the standard deviation of $X$ ?

2012 Question #2

2. A charity fundraiser has a Spin the Pointer game that uses a spinner like the one illustrated in the figure below.




A donation of $2 is required to play the game. For each $2 donation, a player spins the pointer once and receives the amount of money indicated in the sector where the pointer lands on the wheel. The spinner has an equal probability of landing in each of the 10 sectors.

(a) Let $X$ represent the net contribution to the charity when one person plays the game once. Complete the table for the probability distribution of $X$.

| $x$ | $2 | $1 | –$8 |
|---|---|---|---|
| $P(x)$ | | | |

(b) What is the expected value of the net contribution to the charity for one play of the game?

(c) The charity would like to receive a net contribution of \$500 from this game. What is the fewest number of times the game must be played for the expected value of the net contribution to be at least \$500 ?

(d) Based on last year's event, the charity anticipates that the Spin the Pointer game will be played 1,000 times. The charity would like to know the probability of obtaining a net contribution of at least \$500 in 1,000 plays of the game. The mean and standard deviation of the net contribution to the charity in 1,000 plays of the game are \$700 and \$92.79, respectively. Use the normal distribution to approximate the probability that the charity would obtain a net contribution of at least \$500 in 1,000 plays of the game.

2011 Question #2

2. The table below shows the political party registration by gender of all 500 registered voters in Franklin Township.

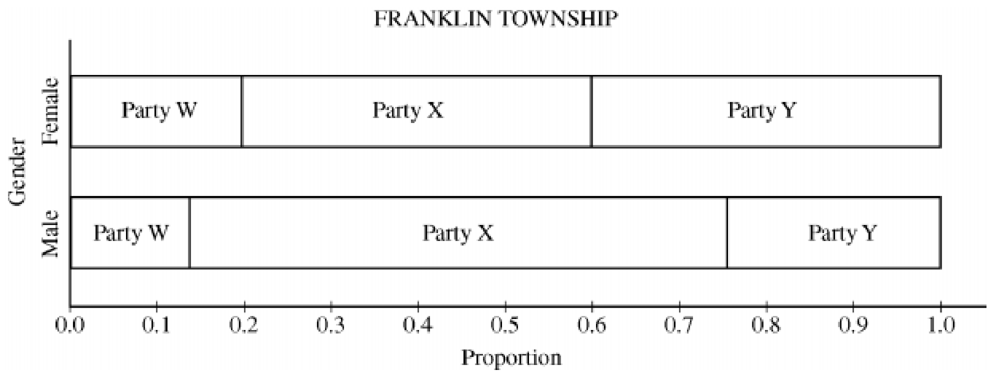
PARTY REGISTRATION–FRANKLIN TOWNSHIP

| | Party W | Party X | Party Y | Total |
|---|---|---|---|---|
| Female | 60 | 120 | 120 | 300 |
| Male | 28 | 124 | 48 | 200 |
| Total | 88 | 244 | 168 | 500 |

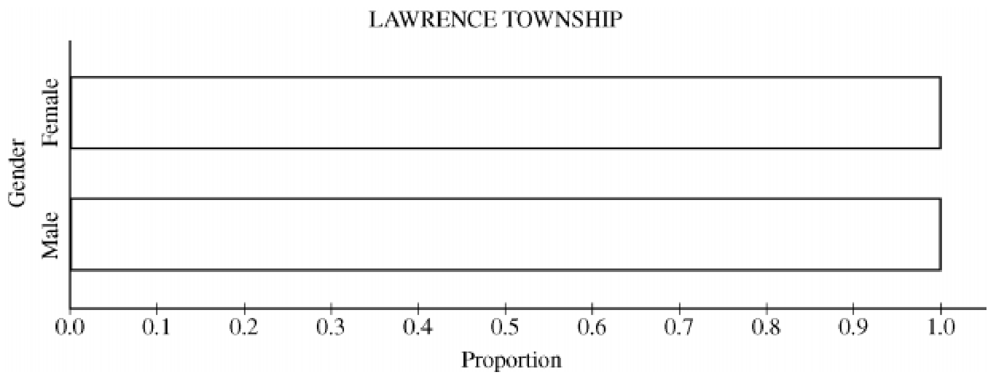
(a) Given that a randomly selected registered voter is a male, what is the probability that he is registered for Party Y?

(b) Among the registered voters of Franklin Township, are the events “is a male” and “is registered for Party Y” independent? Justify your answer based on probabilities calculated from the table above.

(c) One way to display the data in the table is to use a segmented bar graph. The following segmented bar graph, constructed from the data in the party registration–Franklin Township table, shows party-registration distributions for males and females in Franklin Township.




In Lawrence Township, the proportions of all registered voters for Parties W, X, and Y are the same as for Franklin Township, and party registration is independent of gender. Complete the graph below to show the distributions of party registration by gender in Lawrence Township.



2010 Question #4

4. An automobile company wants to learn about customer satisfaction among the owners of five specific car models. Large sales volumes have been recorded for three of the models, but the other two models were recently introduced so their sales volumes are smaller. The number of new cars sold in the last six months for each of the models is shown in the table below.

| Car Model | A | B | C | D | E | Total |
|---|---|---|---|---|---|---|
| Number of new cars sold in the last six months | 112,338 | 96,174 | 83,241 | 3,278 | 2,323 | 297,354 |

The company can obtain a list of all individuals who purchased new cars in the last six months for each of the five models shown in the table. The company wants to sample 2,000 of these owners.

(a) For simple random samples of 2,000 new car owners, what is the expected number of owners of model E and the standard deviation of the number of owners of model E?

(b) When selecting a simple random sample of 2,000 new car owners, how likely is it that fewer than 12 owners of model E would be included in the sample? Justify your answer.

(c) The company is concerned that a simple random sample of 2,000 owners would include fewer than 12 owners of model D or fewer than 12 owners of model E. Briefly describe a sampling method for randomly selecting 2,000 owners that will ensure at least 12 owners will be selected for each of the 5 car models.